PAPER

An SBL-Based Coherent Source Localization Method Using Virtual Array Output

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SUMMARY Direction of arrival (DOA) estimation as a fundamental issue in array signal processing has been extensively studied for many applications in military and civilian fields. Many DOA estimation algorithms have been developed for different application scenarios such as low signal-to-noise ratio (SNR), limited snapshots, etc. However, there are still some practical problems that make DOA estimation very difficult. One of them is the correlation between sources. In this paper, we develop a sparsity-based method to estimate the DOA of coherent signals with sparse linear array (SLA). We adopt the off-grid signal model and solve the DOA estimation problem in the sparse Bayesian learning (SBL) framework. By considering the SLA as a 'missing sensor' ULA, our proposed method treats the output of the SLA as a partial output of the corresponding virtual uniform linear array (ULA) to make full use of the expanded aperture character of the SLA. Then we employ the expectation-maximization (EM) method to update the hyper-parameters and the output of the virtual ULA in an iterative manner. Numerical results demonstrate that the proposed method has a better performance in correlated signal scenarios than the reference methods in comparison, confirming the advantage of exploiting the extended aperture feature of the SLA.

key words: *direction of arrival (DOA) estimation, off-grid model, sparse Bayesian learning (SBL), sparse signal recovery (SSR), coherent source*

1. Introduction

Direction of arrival (DOA) estimation is a crucial task in array signal processing. It has been extensively researched in the past few decades. Many algorithms have already been developed for different scenarios such as low signal-to-noise ratio (SNR), limited snapshots, correlated signal scenarios [1], [2], etc. The most well-known DOA estimation methods are the subspace-based methods represented by MUSIC [3] and ESPRIT [4] which exploit the orthogonality between the signal and noise subspaces of the covariance matrix and have been applied in many fields for their super-resolution and/or low complexity. However, for highly or fully correlated signals, the spatial correlation matrix used in MUSIC (SS-MUSIC) becomes singular or nearly singular, leading to poor estimation performance. For this situation, the authors of [5] proposed a modified version of the MUSIC method with spatial smoothing preprocessing which leads to a full-rank spatially smoothed correlation matrix.

Recently, sparse signal recovery (SSR) methods have been used for DOA estimation [6]. Malioutov etc, proposed an ℓ_p-norm-based method named ℓ_1 reconstruction with singular value decomposition (L1SVD) for DOA estimation [7], where the ℓ_1-norm is used to constrain the sparsity and the SVD is applied to reduce the computation loads. The L1SVD method uses an on-grid signal model to divide the angle space into a set of grids, and assumes that the incident signals exactly lie on the fixed grids. In [8], a so-called sparse iterative covariance-based estimation (SPICE) method is proposed. The authors developed a spatial correlation matching approach by employing the on-grid signal model and extending the spatial correlation matrix. Although SPICE has been proposed for uncorrelated sources, it has been verified to be a robust method as L1SVD in correlated signal scenario.

As the predefined grid set is not continuous, there always exists a bias between the true DOAs and the grids in the on-grid signal model. While reducing the inter-grid space will cause much higher computation, the estimation accuracy is still limited due to the restricted isometry property (RIP) rule [9]. Although the L1SVD method adopts the iterative grid refinement (IGR) strategy to improve the estimation accuracy and reduce the computational complexity, the improvement of estimation accuracy is still limited. Different from the on-grid signal model, the incident signals estimated in the off-grid signal model are no longer restricted to lie on the predefined grid points [10], [11]. Instead, the bias caused by grid mismatch is parameterized approximately by the first order Taylor expansion of array manifold. Many methods have been proposed based on the off-grid signal model. The sparse total least squares (S-TLS) method proposed by Zhu et al. [12] studied the off-grid model for the first time. They exploited the bias of reconstruction matrix with compressed sensing algorithm and then proposed an alternating iteration methodology to solve this non-convex problem. However, this method is not focused on the problem of DOA estimation. Wu et al. [10] proposed a new method named off-grid ℓ_1 Cholesky covariance decomposition (OGL1CCD) which uses a two-step iterative procedure to solve the mismatch problem. The OGL1CCD method can greatly improve the estimation accuracy, but the performance improvement is limited in low SNR and corre-

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lated signals scenarios. In [13], an efficient method for both 1-D and 2-D DOA estimation was proposed by exploiting the Hermitian–Toeplitz structure of the covariance matrix. It can locate more sources than directly using the sample covariance matrix in some cases. Recently, in [14], the authors propose to apply a family of nonconvex penalties on the singular values of the covariance matrix as the sparsity metrics to approximate the rank norm. It can be regarded as a sparsity-based method with the number of sampling grids approaching infinity.

It has been proven that the sparse Bayesian learning (SBL) class of methods have some advantages in DOA estimation. First, these methods do not rely on the RIP rule to guarantee reliable estimation performance and thus facilitate combining some parameters with respect to the priors of the signal. Second, the SBL class of methods have smaller convergence error than the ℓ_p-norm-based methods which means the SBL methods have fewer local minima [15]. Furthermore, the global minima of the SBL class of methods are always the sparsest solution, while the ℓ_p-norm-based and matching pursuit classes of methods do not have this character. More importantly, the SBL based methods are robust to the correlation of the incoming signals. Yang et al. [16] proposed a new method based on off-grid signal model, named as sparse Bayesian inference (OGSBI). It adopts Bayesian inference to estimate the parameters with a Gamma hyperprior assumption as the sparse prior for the signals of interest. Although it also uses SVD procedure to reduce the computation, its estimation accuracy is not satisfactory. Jagannath et al. [17] studied the mismatch problem of DOA estimation in single snapshot case and derived the theoretical Bayesian Cramer-Rao bound (BCRB) for the off-grid signal model. In [11], a perturbed sparsity-based DOA estimation model named perturbed SBL (PSBL) is proposed to solve the mismatch problem. Although it is still an off-grid SBL (OSBL) algorithm, yet different from the off-grid signal model mentioned above, it uses the linear interpolation method to construct an off-grid signal model and obtains an identical estimation precision throughout the whole angle space.

The number of sources that can be estimated is also an important indicator of the performance of the algorithm. However, most of the above mentioned methods focus on increasing the number of identifiable sources by vectorizing the covariance matrix and assuming the sources are uncorrelated [18]. For uncorrelated signal scenarios, some methods have been proposed for DOA estimation where the sources are more than active antennas. Pillai et al. [19] have verified that by using an M-element minimum redundancy array (MRA) and a larger dimension augmented array output covariance matrix, it is possible to estimate as many as $M(M-1)/2$ uncorrelated sources. In [20], the authors use the maximum likelihood method to address the statistical properties of the augmented covariance matrix when the array output covariances are estimated from finite data. Shakeri et al. [21] rewrite the spatial correlation matrix as Khatri-Rao product of the array manifold matrix with grid-based model, and exploit least squares (LS) method to address the DOA estimation problem. A primary technique for expanding the array aperture is based on constructing special array geometries such as the nested array [22], the coprime array [23]–[25], etc. These special array geometries can generally be subsumed into a so-called sparse linear array (SLA), and most of the conventional and SSR-based methods can be adopted for the SLA directly. The biggest advantage of SLA is that it can achieve larger array apertures with the same number of elements as ULA.

The SLA can be regarded as a 'missing sensor' ULA where the inter-sensor spacing can be larger than half wavelength. Due to the enlarged array aperture, the SLA is expected to locate more sources and exhibit higher accuracy than the ULA with the same number of sensors. On the other hand, it can reduce the costs of sensors and other hardware equipments compared to the ULA. Furthermore, the SLA can reduce the mutual coupling between sensors, making the sensor arrangement more flexible.

Based on the characteristic of expanded aperture, in this paper, we present an off-grid method for the estimation of the DOA of correlated sources with the SLA under the SBL framework. The main idea is to treat the output of the SLA as a partial output of the corresponding virtual ULA and then employ the EM method [26], [27] to update the hyperparameters and the output of the virtual ULA with an iterative procedure. Numerical results demonstrate that the proposed method has a better performance than the reference methods in coherent signal scenarios. Specifically, our method can locate 4 coherent signals accurately by an SLA with only 4-elements, where the existing methods would fail.

Notations used in this paper are provided below. Bold symbols denote vectors and matrices. $\boldsymbol{A}^*$, $\boldsymbol{A}^T$, $\boldsymbol{A}^H$ and $\boldsymbol{A}^\dagger$ denote the conjugate, transpose, conjugate transpose and pseudo-inverse of matrix $\boldsymbol{A}$, respectively. vec$(\boldsymbol{A})$ denotes the vectorization operator that stacking A column by column. $\boldsymbol{A}_{(M\times N)}$ denotes the matrix with size $M \times N$. $\boldsymbol{s}_n$ denotes the n-th row of matrix $\boldsymbol{S}$. diag$(\boldsymbol{A})$ denotes the vectorization operator of the entries along the principal diagonal of matrix bmA. tr$(\bullet)$ is the trace operator. $\boldsymbol{I}_N$ denotes the identity matrix with size $N\times N$. $\Re(\bullet)$ denotes the real part of a complex variable. $[K]$ denotes the set of elements $\{1, \cdots, K\}$. $\|\boldsymbol{a}\|_2$ and $\|\boldsymbol{A}\|_F$ denote the ℓ_2-norm and Frobenius norm of vector $\boldsymbol{a}$ and matrix $\boldsymbol{A}$, respectively.

The rest of the paper is organized as follows. Section 2 revisits the signal model as preliminaries. Section 3 introduces our proposed method. Simulations are carried out in Sect. 4 to demonstrate the performance of our method with comparison to several existing approaches. Finally, Sect. 5 concludes the whole paper.

2. Signal Model

Consider that K narrowband far-field signals impinge onto an SLA of D omnidirectional sensors and assume that K is already known. The DOAs of these K signals can be denoted as $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_K]^T$, and then the array output at time t can be expressed as

$$\boldsymbol{x}(t) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t) + \boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t), \cdots, x_D(t)]^T$, $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_K)]$ and $\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)]^T$ are the array output data, the array manifold matrix and incident signals, respectively, and $\boldsymbol{v}(t)$ is a complex white Gaussian noise vector with zero mean which is uncorrelated with source signal vector $\boldsymbol{s}(t)$. Here, $\boldsymbol{a}(\theta_k), k \in [K]$ is the steering vector of the k-th signal and can be denoted as $\boldsymbol{a}(\theta_k) = [e^{-j2\pi d_1 \sin\theta_k/\lambda}, \cdots, e^{-j2\pi d_D \sin\theta_k/\lambda}]^T$, where λ represents the signal wavelength and $d_i, i \in [D]$ denotes the location of the i-th sensor. When L snapshots are collected, the array output can be easily extended to a multiple measurement vector (MMV) model as

$$\boldsymbol{X} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{S} + \boldsymbol{V}, \tag{2}$$

where $\boldsymbol{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(L)]$, $\boldsymbol{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(L)]$, $\boldsymbol{V} = [\boldsymbol{v}(1), \cdots, \boldsymbol{v}(L)]$, $l \in [L]$, respectively. According to the theory of SSR, we uniformly divide the angular space into a fixed set of N possible angles of arrival $\Phi = [\phi_1, \cdots, \phi_N]$ with $N \gg K$, leading to the expanded manifold matrix $\widetilde{\boldsymbol{B}}(\Phi) = [\boldsymbol{b}(\phi_1), \cdots, \boldsymbol{b}(\phi_N)]$. The grid interval is defined as $r = 180^\circ/(N-1)$. In the off-grid signal model, when the incident signal does not lie on the grid set, there exists a bias between the actual source direction θ_k and the nearest grid ϕ_{n_k}, $n_k \in [N]$, denoted as $\delta_{n_k} = \theta_k - \phi_{n_k}$, with $-r/2 \le \delta_{n_k} \le r/2$. In this case, the corresponding steering vector of the k-th signal $\boldsymbol{a}(\theta_k)$ for the off-grid model can be approximated by the first order Taylor expansion

$$\boldsymbol{a}(\theta_k) \approx \boldsymbol{b}(\phi_{n_k}) + \boldsymbol{c}(\phi_{n_k})\delta_{n_k}, \tag{3}$$

where the vector $\boldsymbol{b}(\phi_{n_k})$ is the steering vector corresponding to ϕ_{n_k} and $\boldsymbol{c}(\phi_{n_k})$ is the derivative of $\boldsymbol{b}(\phi_{n_k})$ with respect to ϕ_{n_k}. The bias corresponding to each grid can be denoted as

$$\delta_n = \begin{cases} 0 & \text{otherwise} \\ \theta_k - \phi_{n_k} & \text{if } n = n_k \text{ for any } n \in [N], k \in [K] \end{cases}. \tag{4}$$

By denoting $\boldsymbol{\Delta} = \text{diag}(\boldsymbol{\delta})$ with $\boldsymbol{\delta} = [\delta_1, \cdots, \delta_N]^T$, the array output in (2) can be reformulated as

$$\boldsymbol{X} = \widetilde{\boldsymbol{A}}\bar{\boldsymbol{S}} + \boldsymbol{V} = (\widetilde{\boldsymbol{B}} + \widetilde{\boldsymbol{C}}\boldsymbol{\Delta})\bar{\boldsymbol{S}} + \boldsymbol{V}, \tag{5}$$

where $\widetilde{\boldsymbol{A}} = (\widetilde{\boldsymbol{B}} + \widetilde{\boldsymbol{C}}\boldsymbol{\Delta})$, $\bar{\boldsymbol{S}} = [\bar{s}(1), \cdots, \bar{s}(L)]$, $\widetilde{\boldsymbol{B}}$ is the abbreviation of $\widetilde{\boldsymbol{B}}(\Phi)$, and $\widetilde{\boldsymbol{C}} = [\boldsymbol{c}(\phi_1), \cdots, \boldsymbol{c}(\phi_N)]$. It can be seen that $\bar{\boldsymbol{S}}$ is the extension of $\boldsymbol{S}$ corresponding to Φ with non-zero entries denoting the true sources locations. For $N \gg K$, $\bar{s}(l)$ is a sparse vector.

3. Proposed Algorithm

In this section, we will elaborate our proposed method based on the SBL framework. Since the SLA can be regarded as a 'missing sensor' ULA, the array output $\boldsymbol{X}$ can be regarded as the selected output of an M-element virtual ULA with $D < M$ and $K < M$, i.e., $\boldsymbol{X} = \boldsymbol{P}\boldsymbol{Y}$, where $\boldsymbol{Y}$ is denoted as the output of the virtual ULA, and $\boldsymbol{P} \in \mathbb{C}^{D\times M}$ is a selection matrix, which can be generated by eliminating the zero rows of diagonal matrix diag$(\boldsymbol{p})$. Here, the binary vector $\boldsymbol{p} \in \mathbb{C}^{M\times 1}$ defines the mapping between the virtual ULA and the SLA: if the i-th ($i \in [M]$) element's output of the virtual ULA is selected as the output of the SLA, the corresponding entry of $\boldsymbol{p}$ is set to 1, otherwise 0. Taking 4-element SLA as an example, the relationship between the SLA and virtual ULA is shown in Fig. 1, where symbol • denotes that there is a sensor corresponding to the location, and △ denotes there is no sensor located in this position. To retain the same aperture as the virtual ULA, it is assumed that the SLA always selects the first and last antennas of the corresponding virtual ULA. Thus, the corresponding binary vector can be expressed as $\boldsymbol{p} = [1\ 1\ 0\ 0\ 1\ 0\ 1]^T$. Then, the array output of the virtual ULA can be given as

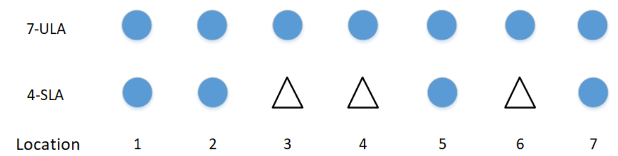

Fig. 1 A 4-element SLA diagram and its relationship with the 7-element ULA.

$$\boldsymbol{Y} = \bar{\boldsymbol{A}}\bar{\boldsymbol{S}} + \boldsymbol{V} = (\bar{\boldsymbol{B}} + \bar{\boldsymbol{C}}\boldsymbol{\Delta})\bar{\boldsymbol{S}} + \bar{\boldsymbol{V}}, \tag{6}$$

where $\bar{\boldsymbol{A}}$ denotes the array manifold of the virtual ULA with $\boldsymbol{P}\bar{\boldsymbol{A}} = \widetilde{\boldsymbol{A}}$, $\boldsymbol{P}\bar{\boldsymbol{B}} = \widetilde{\boldsymbol{B}}$, $\boldsymbol{P}\bar{\boldsymbol{C}} = \widetilde{\boldsymbol{C}}$ and $\boldsymbol{P}\bar{\boldsymbol{V}} = \widetilde{\boldsymbol{V}}$, respectively.

One can achieve the DOA estimate directly by using the output $\boldsymbol{X}$ of the SLA. However, it is expected that more sources could be estimated if we can recover the output of the corresponding virtual ULA according to $\boldsymbol{X}$. It is worth noting that unlike some other methods using second-order statistics to enlarge the degree of freedom (DoF) which led to uncorrelated constraint, we adopt the SBL framework to recover the output of the virtual ULA to enlarge the DoF instead.

We will elaborate our method based on the SBL methodology in the following. The SBL is initially used for regression and classification in machine learning, which is to find the posterior probability $p(\boldsymbol{x}|\boldsymbol{y};\Theta)$ by the Bayesian rule, where Θ denotes the set of all hyperparameters. The hyperparameters can be estimated by marginalizing over $\boldsymbol{x}$ and then performing evidence maximization. Given the hyperparameters, the solution $\hat{\boldsymbol{x}}$ can be obtained by the Maximum-A-Posterior (MAP).

In the off-grid signal model (6), assume that the columns of $\bar{\boldsymbol{S}}$ are mutually independent, and each column is a zero-mean Gaussian vector, i.e.,

$$\bar{s}(t) \sim N(0, \boldsymbol{\Gamma}), t \in [L] \tag{7}$$

where $\boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\gamma})$ is the covariance matrix with $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_N]^T$, $\gamma_n \ge 0$, $n \in [N]$. Note that $\boldsymbol{\Gamma}$ has the same sparse structure with $\bar{\boldsymbol{S}}$ and γ_n is a nonnegative hyperparameter controlling the row sparsity, which means when $\gamma_n = 0$, the associated row of $\bar{\boldsymbol{S}}$ becomes zero. Then we can obtain the probability density function (PDF) of $\bar{\boldsymbol{S}}$ as follows,

$$p(\bar{\boldsymbol{S}};\boldsymbol{\Gamma}) = |\pi\boldsymbol{\Gamma}|^{-L}\exp\left[-tr(\bar{\boldsymbol{S}}^H\boldsymbol{\Gamma}^{-1}\bar{\boldsymbol{S}})\right]. \tag{8}$$

Assume that the entries of the noise matrix $\boldsymbol{V}$ are mutually independent and each row of $\boldsymbol{V}$ has a complex Gaussian distribution, i.e.,

$$v(t) \sim N(0, \sigma\boldsymbol{I}), t \in [L], \tag{9}$$

where σ is the noise power. For the MMV model (6) the Gaussian likelihood is given by

$$p(\boldsymbol{Y}|\bar{\boldsymbol{S}};\sigma,\boldsymbol{\delta}) \sim N(\bar{\boldsymbol{A}}\bar{\boldsymbol{S}}, \sigma\boldsymbol{I}). \tag{10}$$

In our signal model, the output $\boldsymbol{Y}$ of equivalent virtual ULA is unknown. Considering the relation between $\boldsymbol{Y}$ and $\boldsymbol{X}$, the mean of $p(\boldsymbol{Y}|\boldsymbol{X})$ can be given as

$$\boldsymbol{\mu}_{\boldsymbol{Y}|\boldsymbol{X}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{S}} + \boldsymbol{P}^H(\boldsymbol{X} - \boldsymbol{P}\bar{\boldsymbol{A}}\bar{\boldsymbol{S}}). \tag{11}$$

Using the Bayes rule we obtain the posterior PDF of $\bar{\boldsymbol{S}}$ as

$$p(\bar{\boldsymbol{S}}|\boldsymbol{Y};\boldsymbol{\Gamma},\sigma,\boldsymbol{\delta}) \sim N(\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}, \boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}), \tag{12}$$

with mean and covariance matrix being given by

$$\begin{aligned} \boldsymbol{\mu}_{\bar{\boldsymbol{S}}} &= \boldsymbol{\Gamma}\bar{\boldsymbol{A}}^H\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\boldsymbol{Y} \\ \boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}} &= (\boldsymbol{\Gamma}^{-1} + \sigma^{-1}\bar{\boldsymbol{A}}^H\bar{\boldsymbol{A}})^{-1} \;= \boldsymbol{\Gamma} - \boldsymbol{\Gamma}\bar{\boldsymbol{A}}^H\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\bar{\boldsymbol{A}}\boldsymbol{\Gamma}, \end{aligned} \tag{13}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{Y}} = \sigma\boldsymbol{I} - \bar{\boldsymbol{A}}\boldsymbol{\Gamma}\bar{\boldsymbol{A}}^H$.

Due to the strict convergence characteristic of the EM method, it is used to maximize the posterior $p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\boldsymbol{\Theta})$ in order to find the hyperparameters $\Theta = \{\boldsymbol{\gamma}, \boldsymbol{\delta}, \sigma\}$ while treating $\bar{\boldsymbol{S}}$ as a hidden variable. This is equivalent to maximizing the log-posterior $\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\boldsymbol{\Theta})$. The EM method consists of two steps: the expectation (E) step and the maximization (M) step. The E-step computes the expected value of $\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\boldsymbol{\Theta})$ which is defined as the Q-function. Given the virtual array output $\boldsymbol{Y}$ and the estimated hyperparameters Θ^{old} from the previous iteration, the Q-function is computed by

$$\begin{aligned} Q(\Theta) &= E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\Theta)] \\ &= E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\sigma,\boldsymbol{\delta})p(\bar{\boldsymbol{S}};\boldsymbol{\Gamma})], \end{aligned} \tag{14}$$

Note that in computing the Eq. (14), $p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\sigma,\boldsymbol{\delta})$ does not depend on $\boldsymbol{\Gamma}$, and $p(\bar{\boldsymbol{S}};\boldsymbol{\Gamma})$ is independent of $\{\sigma, \boldsymbol{\delta}\}$. Hence, Eq. (14) can be decomposed as

$$Q(\Theta) = E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\sigma,\boldsymbol{\delta})] + E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[\log p(\bar{\boldsymbol{S}};\boldsymbol{\Gamma})]. \tag{15}$$

Next, the M-step is used to maximize the above Q-function to find a new estimate of Θ. To estimate $\boldsymbol{\Gamma}$, Eq. (15) can be simplified to $Q(\boldsymbol{\Gamma}) = E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[-\log p(\bar{\boldsymbol{S}};\boldsymbol{\Gamma})]$. Since $\bar{\boldsymbol{S}}$ is Gaussian, $Q(\boldsymbol{\Gamma})$ can be denoted as

$$\begin{aligned} Q(\boldsymbol{\Gamma}) &= E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}[L\log|\pi\boldsymbol{\Gamma}| + \text{tr}(\bar{\boldsymbol{S}}^H\boldsymbol{\Gamma}^{-1}\bar{\boldsymbol{S}})] \\ &\simeq L\log(|\boldsymbol{\Gamma}|) + L\text{tr}[\boldsymbol{\Gamma}^H(\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}} + \boldsymbol{\mu}_{\bar{\boldsymbol{S}}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}^H)]. \end{aligned} \tag{16}$$

The derivative of formula (16) with respect to $\gamma_n, n \in [N]$ can be expressed as

$$\frac{\partial Q(\boldsymbol{\Gamma})}{\partial\gamma_n} = -\frac{L}{\gamma_n} + \frac{L}{\gamma_n^2}\left[(\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}})_{n,n} + \frac{\|(\boldsymbol{\mu}_{\bar{\boldsymbol{S}}})_{n,:}\|_2^2}{L}\right]. \tag{17}$$

Letting (16) be zero, the iterative expression of γ_n can be deduced as follows,

$$\gamma_n = (\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}})_{n,n} + \frac{\|(\boldsymbol{\mu}_{\bar{\boldsymbol{S}}})_{n,:}\|_2^2}{L}. \tag{18}$$

Similarly, (15) can be simplified as follows to estimate σ and $\boldsymbol{\delta}$

$$\begin{aligned} Q(\sigma,\boldsymbol{\delta}) &= E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}\left[\log p(\boldsymbol{Y}, \bar{\boldsymbol{S}};\sigma,\boldsymbol{\delta})\right] \\ &= E_{\bar{\boldsymbol{S}}|\boldsymbol{Y};\Theta^{\text{old}}}\left[ML\log|\pi\sigma| + \sigma^{-1}\|\boldsymbol{Y} - \bar{\boldsymbol{A}}\bar{\boldsymbol{S}}\|_F^2\right] \\ &\simeq ML\log\sigma + \sigma^{-1}\Big\{\|\boldsymbol{Y} - \bar{\boldsymbol{A}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}\|_F^2 \\ &+ E_{\bar{\boldsymbol{S}}|\boldsymbol{X};\Theta^{\text{old}}}[\|\bar{\boldsymbol{A}}(\bar{\boldsymbol{S}} - \boldsymbol{\mu}_{\bar{\boldsymbol{S}}})\|_F^2]\Big\} \\ &= ML\log\sigma + \sigma^{-1}\Big[\|\boldsymbol{Y} - \bar{\boldsymbol{A}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}\|_F^2 \\ &+ L\text{tr}(\bar{\boldsymbol{A}}\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}\bar{\boldsymbol{A}}^H)\Big]. \end{aligned} \tag{19}$$

Letting (19) be zero with respect to the derivative of σ, the iterative expression of σ can be given as

$$\sigma = \frac{\|\boldsymbol{Y} - \bar{\boldsymbol{A}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}\|_F^2 + L\text{tr}(\bar{\boldsymbol{A}}\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}\bar{\boldsymbol{A}}^H)}{ML}. \tag{20}$$

Then, by bringing (6) into (19) and following the derivation of [16], the iteration rule of $\boldsymbol{\delta}$ can be expressed as

$$\boldsymbol{\delta} = \boldsymbol{U}^{-1}\boldsymbol{G}, \tag{21}$$

where

$$\boldsymbol{U} = \Re\left\{(\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}^H + L\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}) \circ (\bar{\boldsymbol{C}}^H\bar{\boldsymbol{C}})\right\} \tag{22}$$

$$\boldsymbol{G} = \Re\left\{\text{diag}[\bar{\boldsymbol{C}}^H(\boldsymbol{Y} - \bar{\boldsymbol{B}}\boldsymbol{\mu}_{\bar{\boldsymbol{S}}})\boldsymbol{\mu}_{\bar{\boldsymbol{S}}}^H - L\bar{\boldsymbol{C}}^H\bar{\boldsymbol{B}}\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}]\right\}. \tag{23}$$

When EM method converges, the final estimates of signals power spectrum and the corresponding angle bias can be obtained as $\hat{\boldsymbol{\gamma}}^{\text{final}}$ and $\hat{\boldsymbol{\delta}}^{\text{final}}$, respectively. Then the final DOA estimates can be given as

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}^{\text{final}} + \hat{\boldsymbol{\delta}}^{\text{final}}, \tag{24}$$

where $\hat{\boldsymbol{\phi}}^{\text{final}}$ denotes the angles corresponding to the maximum K peaks of $\hat{\boldsymbol{\gamma}}^{\text{final}}$. Finally, our proposed method can be summarized in Algorithm 1 bellow.

4. Numerical Results

In this section, we evaluate the performance of the proposed

Algorithm 1 Proposed method

Input: $\boldsymbol{X}, K, N, \boldsymbol{p}$.
Initialization: $\sigma^{(0)}, \boldsymbol{\gamma}^{(0)}, \boldsymbol{\delta}^{(0)}, \bar{\boldsymbol{S}}$.
repeat
(1)Update $\boldsymbol{Y}$ by equation (11);
(2)Update $\mu_{\bar{\boldsymbol{S}}}$ and $\boldsymbol{\Sigma}_{\bar{\boldsymbol{S}}}$ by equation (13);
(3)Update γ_n by equation (18);
(4)Update noise power σ by equation (20);
(5)Update angle bias $\boldsymbol{\delta}$ by equation (21);
until Convergence
Output: $\hat{\boldsymbol{\theta}}$ by using (24).

method with comparison to L1-SVD, SS-MUSIC [5] and off-grid SBL (OSBL) as well as the stochastic Cramer-Rao Lower Bound (CRLB) by extensive simulations. The L1-SVD method is a classical method of DOA estimation in SSR field which is based on on-grid signal model. It has the advantage of low computation and good robustness to coherent signal scenarios. The SS-MUSIC is a subspace-based and widely used traditional method especially for coherent signal. To make a fair comparison of the proposed method with the reference methods, we first set simulation conditions and definitions. We consider an SLA with 4 sensors represented by vector $\boldsymbol{p} = [1\ 1\ 0\ 0\ 1\ 0\ 1]^T$, and assume the leftmost sensor as the start position. In the initialization of our proposed method, let $\boldsymbol{\gamma}^{(0)} = \frac{1}{ML}\Sigma_{l=1}^{L}|\bar{\boldsymbol{B}}^H \boldsymbol{X}(l)|$, $\sigma^{(0)} = 0.1 \times \|\boldsymbol{X}\|_F^2/(ML)$, $\boldsymbol{\delta}^{(0)} = \boldsymbol{0}$. The stop criterion is given by

$$\frac{|\boldsymbol{\delta}^{(n+1)} - \boldsymbol{\delta}^{(n)}|}{\boldsymbol{\delta}^{(n)}} < 10^{-4}. \tag{25}$$

The root mean square error (RMSE) is defined as

$$\text{RMSE} = \sqrt{\frac{1}{\bar{N}K}\sum_{n=1}^{\bar{N}} \|\hat{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_n\|_2^2}, \tag{26}$$

where $\bar{N}$ denotes the total number of trials, $\hat{\boldsymbol{\theta}}_n$ and $\boldsymbol{\theta}_n$ the estimated and true directions of signals in the n-th trial. The maximum number of iterations is set to 3000. All simulations are carried out in MATLAB 2017a on a PC with a Windows 7 system and a quad core 3.2G CPU.

Our proposed method mainly focuses on DOA estimation in the coherent signal scenario. First, we compare the performance of our proposed method with that of other methods for different grid intervals under two coherent signals. Assume that the two coherent signals impinge onto the SLA from $[-10^\circ + z, 5^\circ + z]$ and let $L = 100$, SNR = 5 dB. The variable z is a random variable with uniform distribution chosen from $[-r/2, r/2]$ and is used to avoid the true DOA landing on the grid points exactly. We chose five different grid intervals, i.e., $r = 1^\circ, 2^\circ, 3^\circ, 4^\circ, 5^\circ$, and carry out 300 independent Monte Carlo trials. The RMSE and CPU time are shown in Fig. 2 and Fig. 3, respectively.

From Fig. 2, it can be seen that, in addition to the SS-MUSIC algorithm, the RMSE of other methods are getting worse as grid interval increases, and our proposed method is closer to the CRLB curve. From Fig. 3, it is seen that other than L1SVD, the computational complexity falls as grid interval rises. The computational complexity of our method and OSBL declines slowly when the grid interval exceeds 2°. Considering the balance between computation and estimation accuracy, we choose the grid interval $r = 2^\circ$. It is noted that, when $r = 2^\circ$, L1SVD, OSBL and our proposed method have almost the same computational complexity.

Then, we compare the RMSE of those methods on different SNRs. The basic settings are the same as that in Fig. 4 and we set the SNR from −15 dB to 15 dB. As shown in Fig. 4, when SNR < −10 dB, the performance of our proposed method and OSBL are almost the same, but when SNR ≥ −10 dB, our proposed method is much better than others. Especially, it shows the effectiveness of our idea by making a comparison with OSBL method. The numerical results show that when SNR ≥ −3 dB, the performance of our method have an improvement from 2.3 dB to 4.3 dB than the OSBL method. Specifically, when SNR = −6 dB, −3 dB and 9 dB, the POSBL gives an improvement of 0.6 dB,

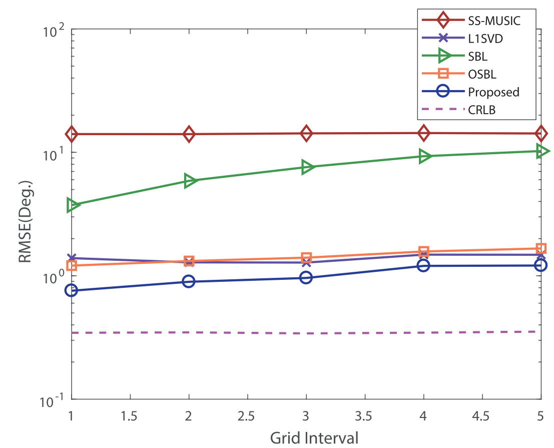

Fig. 2 RMSE comparisons for different methods in terms of grid interval with two coherent signals impinging onto a 4-SLA from $(-10^\circ + z, 5^\circ + z)$. $L = 100$. SNR = 5 dB.

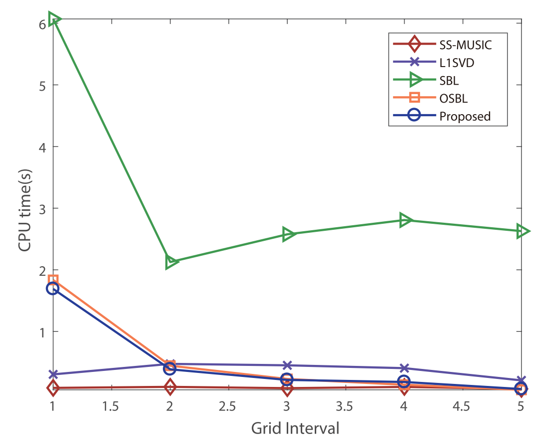

Fig. 3 Time comparisons for different methods in terms of grid interval with two coherent signals impinging onto a 4-SLA from $(-10^\circ + z, 5^\circ + z)$. $L = 100$. SNR = 5 dB.

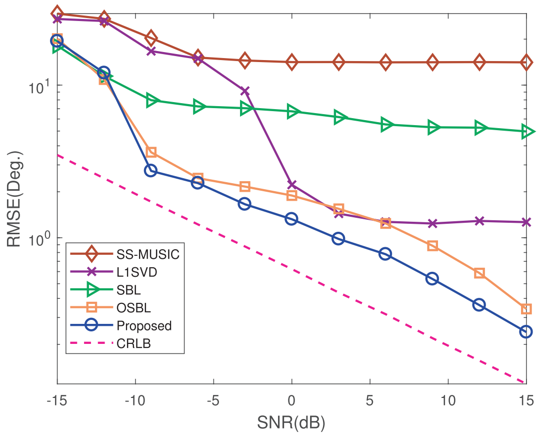

Fig. 4 RMSE comparisons of different methods with two coherent signals impinging onto a 4-SLA from $(-10^\circ + z, 5^\circ + z)$. $L = 100$ in terms of SNR.

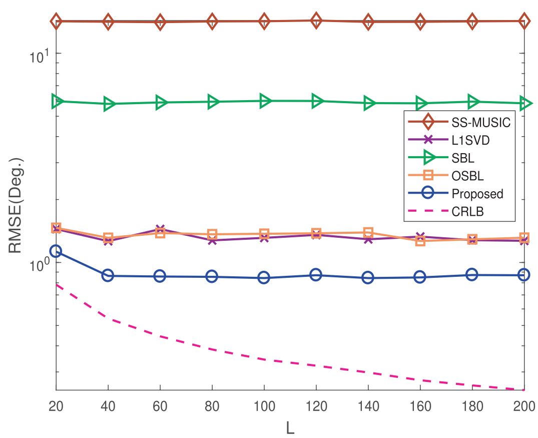

Fig. 5 RMSE comparisons for different methods with two coherent signals impinging onto a 4-SLA from $(-10^\circ + z, 5^\circ + z)$ in terms of the number of snapshots, SNR = 5 dB.

2.3 dB and 4.3 dB than the OSBL method respectively.

Next, we compare the performance with fixed SNRs but different number of snapshots. Assume that SNR = 5 dB, and two coherent signals impinge onto the SLA from $[-10^\circ + z, 5^\circ + z]$. The simulation results are shown in Fig. 5. It can be seen that our proposed method has a better performance than all other methods, and it is worth noting that the performance of all methods almost have no improvement while $L \geq 40$. This observation shows that increasing the number of snapshots can not provide obvious performance improvement in coherent signals scenarios.

Finally, we test the capacity of estimating multiple coherent sources with our proposed method. Let SNR = 5 dB, $L = 100$ and assume 4 coherent signals impinge onto the SLA from $[-32^\circ, -10^\circ, 5^\circ, 25^\circ]$. The spectrum of our proposed method is shown in Fig. 6. It can be observed that our method can locate all the 4 coherent sources accurately with 4-element SLA.

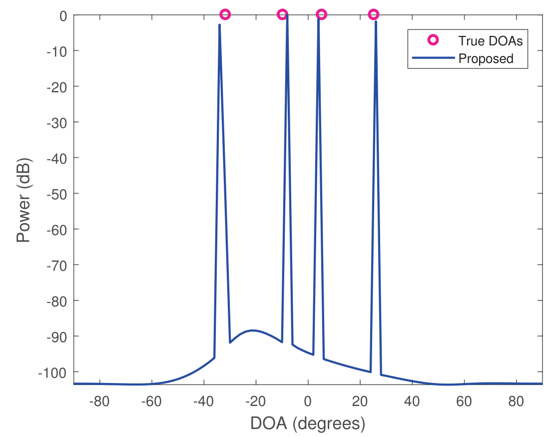

Fig. 6 Spectrum of our method with 4 coherent signals impinging onto a 4-SLA from $(-32^\circ, -10^\circ, 5^\circ, 25^\circ)$. $L = 100$. SNR = 5 dB.

5. Conclusion

In this paper, we have presented an off-grid DOA estimation method in SBL framework. By using a selection matrix, the output of the SLA is converted into an output of the corresponding virtual ULA. The EM method is employed to update the hyper-parameters and the output of the virtual ULA iteratively. Numerical results demonstrate that the proposed method has a better performance in correlated signal scenarios than several other methods in literature. Especially, when SNR ≥ -3 dB, our method brings an improvement from 2.3 dB to 4.3 dB than the OSBL method. Thus, the effectiveness of our proposed method has been verified.

Acknowledgments

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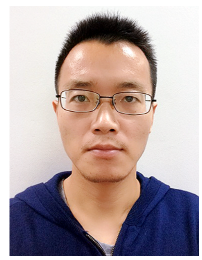

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