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Possibilities and Challenges of Superconducting Qubits in the Intrinsic Josephson Junctions

Haruhisa KITANO[†a)], ***Nonmember***

SUMMARY Intrinsic Josephson junctions (IJJs) in the high-T_c cuprate superconductors have several fascinating properties, which are superior to the usual Josephson junctions obtained from conventional superconductors with low T_c, as follows; (1) a very thin thickness of the superconducting layers, (2) a strong interaction between junctions since neighboring junctions are closely connected in an atomic scale, (3) a clean interface between the superconducting and insulating layers, realized in a single crystal with few disorders. These unique properties of IJJs can enlarge the applicable areas of the superconducting qubits, not only the increase of qubit-operation temperature but the novel application of qubits including the macroscopic quantum states with internal degree of freedom. I present a comprehensive review of the phase dynamics in current-biased IJJs and argue the challenges of superconducting qubits utilizing IJJs.

key words: *intrinsic Josephson junction, phase switching event, nonlinear bifurcation, superconducting qubit*

1. Introduction

Josephson junction (JJ) is a weakly-linked structure of two superconductors, realized by the deposition of a thin insulating (or normal-metallic) layer and the fabrication of a superconducting microbridge [1]. The basic properties of JJs are well explained by Josephson effects, obtained from the macroscopic wavefunction representing the quantum condensation state of Cooper pairs. Josephson devices have been used as the fundamental devices of the superconducting electronics, such as SQUID realizing ultrasensitive measurement and QPU (Quantum Processor Unit) implementing Josephson qubits. At present, many of commercially-available Josephson devices are artificially produced by the microfabrication of thin films of conventional superconductor, Al or Nb. The physical properties of such JJs are so precisely controlled that the superconducting electronic circuits can be designed in a similar manner to usual electronic circuits.

On the other hand, the intrinsic Josephson junctions (IJJs) were discovered in the high-T_c cuprate superconductors, as a unique structure where JJs were naturally embedded into the crystal structure of layered superconductors [2]. In the cuprate superconductors, a block of the superconducting CuO_2 layer and that of the transition-metal oxide layer, playing a role of charge reservoir, are alternately stacked along the c axis in the crystal structure. The

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[†]The author is with Dept. of Physics, Aoyama Gakuin University, Sagamihara-shi, 252–5258 Japan.
a) E-mail: hkitano@phys.aoyama.ac.jp
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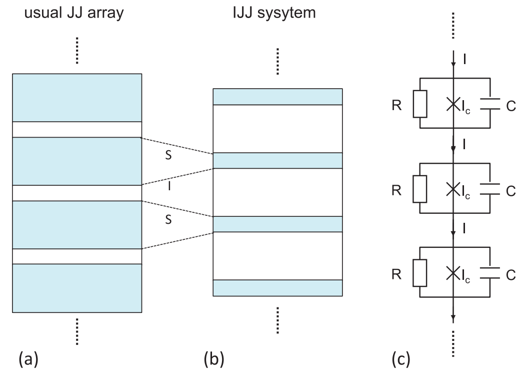

Fig. 1 Schematics of (a) usual Josephson junction array and (b) intrinsic Josephson junction system. (c) Equivalent circuit of Josephson junction array based on the resistively and capacitively shunted junction (RCSJ) model.

carrier doping to CuO_2 layers is usually performed by the substitution of the transition metal element or the control of the oxygen content, leading to a drastic change of the electronic phases from antiferromagnetic insulator to the high temperature superconductivity surviving up to a boiling point of liquid nitrogen. While the intrinsic Josephson effects are also observed in other layered superconductors such organic superconductors and iron based superconductors, the distinct feature of IJJs in the cuprate system is the strong two-dimensional nature emerged in the electronic state of carrier-doped CuO_2 planes. Particularly, in IJJs of $Bi_2Sr_2CaCu_2O_y$ (Bi2212), a good insulating property in BiO layer block permits the formation of one-dimensional array of tunnel-type JJs (that is, SIS junction) in the crystal structure. This suggests that the underdamped JJs used in Josephson qubit applications are intrinsically prepared in Bi2212 single crystals. As shown in Figs. 1 (a) and 1 (b), the important features of Bi2212-IJJs are

(1) The thickness of the superconducting layer block is very small (typically, ~0.3 nm).
(2) The thickness of the insulating block is also very small (~1.2 nm), leading to the strong interaction between JJs.
(3) The critical current density j_c of IJJs is much larger (up to several kA/cm^2) than that for other artificial JJs made of the same Bi2212 films, since an ideal interface without any disorder can be made between the superconducting CuO_2 layer and the insulating BiO layer in the crystal growth.

Here, the first and second points are very interesting as the physical origin of novel phenomenon which will be discovered in IJJs. The third point is very attractive in terms of the increase of an operating temperature of Josephson qubits, as discussed below.

Unfortunately, at present, the implementation of Josephson qubits in IJJs has not been achieved yet. In this paper, we begin with the comprehensive description of the phase dynamics in the current-biased IJJs, featuring the observations of the macroscopic quantum tunneling (MQT) and the energy level quantization (ELQ). Next, we discuss the resonant phase escapes from the finite voltage state, where the temporally-oscillating Josephson current gives an important contribution. Finally, we argue about a possible breakthrough toward the implementation of Josephson qubits utilizing IJJs and a possible application of IJJ-based qubits with internal degree of freedom.

2. MQT and ELQ in the Current-Biased IJJs

2.1 Phase Escapes from the Zero-Voltage State

It is well known that the current-voltage (I-V) characteristics of IJJs have the so-called *multiple-branch* structure with the successive voltage jumps, as shown in Fig. 2 (a). Each voltage jump suggests that one of JJs included in a stack of IJJs is switched into the voltage state, since the bias current exceeds a critical current of the JJ. When each junction of IJJs ideally has the same critical current, as shown in Fig. 2 (b), all junctions in the IJJs are simultaneously switched into the voltage state (*uniform* switch). Thus, the magnitude of the voltage jump in the I-V curves becomes a rough measure of the superconducting gap or a check of the coinstantaneous switch in several JJs. Note that this is true for the case of *small* junction where the lateral sizes of JJ is smaller than the Josephson penetration depth, λ_J. Because of the small thickness of the superconducting layers in IJJs, the magnitude of λ_J for IJJs is reduced down to ~1 μm [2]. In the case of *large* (or *long*) junctions where the sizes of JJ is larger than λ_J, the contribution of Josephson vortices, generated by the self-field of the c-axis current flow and penetrating into the insulating BiO layer, must be considered, giving a more complicate behavior to the I-V characteristics [3].

In order to avoid such complicated influence due to the Josephson vortices, a small stack of IJJs with a lateral area smaller than $1 \times 1\,\mu\mathrm{m}^2$ has been prepared by employing the following two device structures. One is a mesa structure shaped on the surface of Bi2212 single crystals, which is similar to the device structure of IJJ-based THz wave emitters, as shown in Fig. 3 (a). The other is the bridge-type IJJs, snicked by two slits on a narrow microbridge of Bi2212 single crystals, as shown in Fig. 3 (b). These structures are fabricated by using the Ar ion milling or the focused ion beam (FIB) etching. The number of JJs involved in such stacks ranges from a few to several dozen. In the case of the mesa-type IJJs, the value of j_c for the first voltage jump is always reduced by the quasiparticle injection from the normal-metallic electrode covering the top of the mesa. Thus, the position of the first junction switching to the voltage state can be easily identified. On the other hand, in the bridge-type IJJs, it is very difficult to specify the firstly-switched junction. However, the standard four-terminal configuration is available for the bridge-type IJJs, in contrast to the mesa-type IJJs with the three-terminal configuration.

The first observation of MQT in IJJs, reported by Inomata *et al.* [4], was established for the first switch from the zero-voltage state of the bridge-type Bi2212-IJJs fabricated by using the FIB techniques. The large value of j_c ($> 3.3\,\mathrm{kA/cm^2}$) in IJJs has an advantage in the increase of the zero-bias Josephson plasma frequency ω_{p0} ($\sim 2\pi \times$ 220 GHz), which is at least an order of magnitude larger than that for the conventional JJs. This enhances a crossover temperature T_{cr} to the MQT state up to ~1 K, which is proportional to the current-dependent Josephson plasma frequency, as shown in Fig. 4 (a),

$$T_{\mathrm{cr}} \propto \omega_{\mathrm{p}} = \omega_{\mathrm{p0}}[1 - (I/I_c)^2]^{1/4}. \quad (1)$$

This result is quite surprising, since the nodal quasiparticles in d-wave superconductor is expected to prevent the MQT state. Actually, in the *in-plane* type gain boundary JJs made of $\mathrm{YBa_2Cu_3O_y}$ (YBCO) biepitaxial films, the observed value of T_{cr} was reduced down to ~40 mK [5], proving that the MQT state was strongly influenced by the dissipation due to nodal quasiparticles, as suggested by the Caldeira-Leggett theory [6]. In contrast, the first-principle band calculation for cuprates suggested that the transfer integral along the *out-of-plane* direction was strongly suppressed for the nordal quasiparticles with $k_y = \pm k_x$, as follows [7],

$$t_{\perp}(k_x, k_y) \propto (\cos k_x a - \cos k_y a)^2. \quad (2)$$

Here, a is a lattice constant of $\mathrm{CuO_2}$ plane in the tetragonal phase. This feature excellently explains not only the

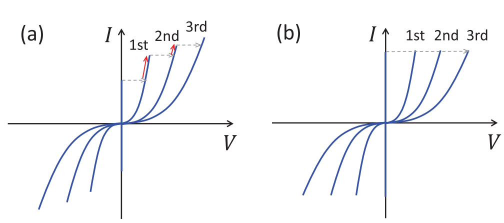

Fig. 2 I-V characteristics of IJJs, (a) a typical behavior, (b) an ideal case of IJJs where each junction has the same I_c. Red arrows in (a) represent the phase running state in the phase switched junction.

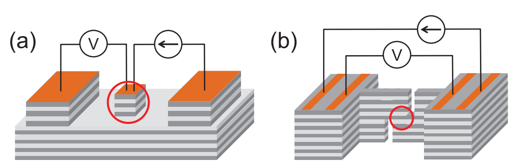

Fig. 3 Schematics of (a) mesa-type IJJs and (b) bridge-type IJJs together with a current source and a voltmeter. Red circles represent active IJJs.

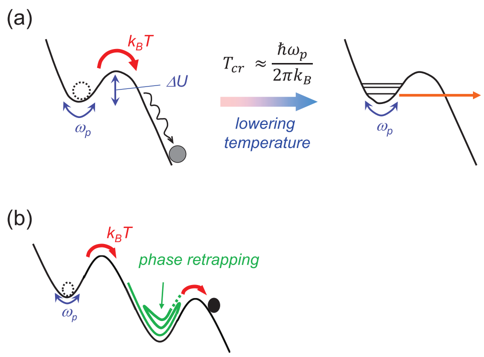

Fig. 4 A fictitious phase particle escaping from the potential well of tilted washboard potential. (a) a crossover from the thermally activated (TA) phase escapes to the macroscopic quantum tunneling (MQT) with lowering temperatures. (b) TA escape and phase retrapping (PR) processes.

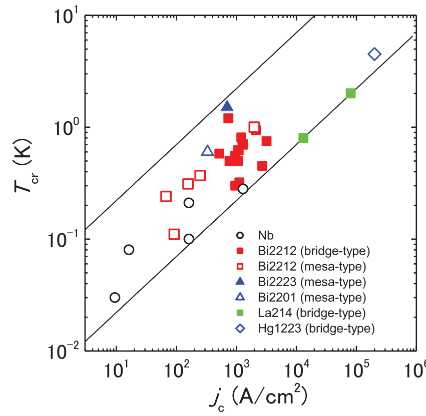

Fig. 5 Comparative plots of T_{cr} versus j_c between several IJJs and Nb-based JJs, which were reported by the previous studies. For data of IJJs, only the phase switches from the zero-voltage state, that is, the 1st SWs and the uniform SWs are used, while typical values are used for Nb-JJs. Black circles are Nb-JJs ([20], [21]). Red solid (open) squares are the mesa-type (bridge-type) Bi2212-IJJs ([4], [10], [12], [14], [15], [37], [38]). Blue solid (open) triangles are the mesa-type Bi2223 (Bi2201)-IJJs ([18], [38]). Green solid squares are the bridge-type La214-IJJs ([17], [19]). Blue open diamond is the bridge-type Hg1223-IJJs ([16]). Solid lines are guides to eyes, showing $T_{cr} \propto \sqrt{j_c}$.

anomalously anisotropic transport in the normal- and superconducting states [8], but also the dramatic decrease of the nodal-quasiparticle damping in the MQT state [9]. Thus, among the cuprate-based JJs, the IJJ is the ideal structure where the MQT state can be preserved in spite of the existence of quasiparticles.

The formation of ELQ was confirmed by the microwave irradiation experiments. In the MQT state, the Schrödinger's equation gives the quantized eigenvalues for a fictitious phase particle in the potential well of tilted washboard potential, as shown in Fig. 4 (a). Thus, the microwave absorption corresponding to the energy-level spacing increases the population of the excited state and the phase escape from the excited state. Jin *et al.* [10] reported the microwave-induced phase escapes from the zero-voltage state, through the multiphoton absorption process, satisfying a relation that $\omega_p = n\omega$ (Here, $n = 2$ or 3, ω is the irradiated microwave frequency). Note that $\hbar\omega_p$ is nearly equal to the energy spacing between the ground state and the first excited state. Similar experimental results were also obtained by several groups ([11]–[13]).

The confirmation of MQT and ELQ in the zero-voltage state of IJJs had been almost completed until the late 2000s. Figure 5 shows a summary of T_{cr} as a function of j_c, where each plot was obtained for the phase escapes from the zero-voltage state of the IJJs made of several cuprate superconductors ([4], [10], [12], [14]–[16], [18]) and some of the conventional Nb-based JJs ([20], [21]). The plots of T_{cr} versus j_c roughly show a trend that $T_{cr} \propto \sqrt{j_c}$, although there is a broad distribution in the values of T_{cr} obtained for a fixed value of j_c. The trend of T_{cr} increasing with $\sqrt{j_c}$ is expected by the fact that ω_{p0} is determined by $\sqrt{j_c}$. On the other hand, the distribution of T_{cr} has a width of about an order of magnitude in the log-log plots, which is larger than that for the conventional JJs. Thus, the broad distribution of T_{cr} observed in IJJs suggests that T_{cr} is not determined by only j_c. For a single JJ, it is well known that T_{cr} is dependent on the bias current normalized by I_c and the damping factor $1/Q$, as well as ω_{p0} [20]. Interestingly, in the IJJs showing a *uniform* switch where all junctions were coinstantaneously switched, the enhancement of T_{cr} was reported [10]. Although several theories were proposed to explain such enhancement ([22]–[24]), the distributed behavior of T_{cr} observed in IJJs remains unresolved.

2.2 Phase Escapes from the Finite Voltage State

The strong interaction between nearest neighbor JJs is the unique property of IJJs, as emphasized in Sect. 1. The classical interaction such as an inductive and capacitive coupling between JJs has been discussed for not only IJJs but an array of the conventional JJs [25]–[27]. Thus, the occurrence of the MQT and ELQ is expected for the phase switching processes from the finite voltage state, which occurs after the first switch (1st SW) from the zero-voltage state. Kashiwaya *et al.* reported that the MQT-like behavior for the second switch (2nd SW) from the 1st voltage state of the bridge-type IJJs was maintained up to an anomalously high temperature T^*(~ 6.5 K) [15].

In the conventional model representing a resistively and capacitively shunted junction (RCSJ model), the same bias current is assumed to flow in each junction of the JJ array, as shown in Fig. 1 (c). If there is no difference in the properties of each JJ, it is expected that the value of T_{cr} to the MQT state is independent of the order of the phase switch. Actually, the switching current for the 2nd SW was distributed around a slightly smaller value than that for the 1st SW, suggesting that T_{cr} for the 2nd SW is lower than that for the 1st SW. However, the value of T^* was found to be much higher than the calculated values of T_{cr} for both the 1st and 2nd SWs. Thus, the authors of Ref. [15] concluded that such anomalous behavior was due to the Joule heating generated in the switched junction.

In order to test the self-heating effects, Ota *et al*. compared the 2nd SWs for the bridge-type IJJs with those for the mesa-type IJJs [12]. Note that there is a distinct difference in the heat transfer environment around the IJJs between both device structures. As shown in Fig. 3, the mesa-type IJJs can efficiently remove the Joule heating generated in the voltage state through the capping electrode on the top surface of IJJs and the bulk crystal beneath the IJJs, while the bridge-type IJJs are isolated by two slits and are far away the remaining part of bulk crystal and the electrode, implying insufficient heat diffusion. Thus, the value of T^* observed for the 2nd SW is expected to be quite different between both device structures, if T^* was influenced by the self-heating effects. However, the obtained results could not be explained by the self-heating effects, since the values of $T^*(\sim 8\,\mathrm{K})$ was common to both structures.

Similar behaviors were also observed for the higher order phase switch than the 2nd SW. Kakehi *et al*. investigated the phase switches from the 1st SW to 4th SW of the bridge-type IJJs with only 10 junctions [28]. They analyzed the measured results of the switching current distrbution $P(I)$ by fitting to the calculated $P(I)$ by considering the conventional thermally-activated (TA) phase escapes and the phase retrappings (PR) succeedingly occurring after the phase escape events, as shown in Fig. 4 (b). They obtained an effective temperature T_{eff} for each order of SW as a function of bath temperature. Although the lowest temperature of the measurements was 4 K, T_{eff} shows a clear saturation for bath temperature below ~8 K, except for the 1st SW. They also found that T_{eff} for the 3rd and the 4th SWs is smaller than that for the 2nd SW. This behavior can not be explained by the self-heating scenario, since the quantity of Joule heating is expected to scale up with the number of the junction switched to the voltage state. Thus, the MQT-like behavior observed for the higher order SWs needs other scenario than the self-heating effect.

The observation of ELQ can be strong evidence for the occurrence of the MQT state for the higher order SWs, as like as the 1st SWs. Unfortunately, Ota *et al*. could not find any resonant feature for the 2nd SWs at the lowest temperature (∼ 0.4 K), although they attempted to measure $P(I)$ near the microwave frequencies where the ELQ for the 1st SWs was confirmed [12]. The resonant escapes for the 2nd SWs under the strong microwaves was reported by Takahashi *et al*. [29]. Figure 6 (a) shows schematic examples of such resonant escapes induced by microwave irradiation. As shown in Fig. 6 (c), the microwave power dependence of the most frequently-switching current $I_s(P_{\mathrm{MW}})$ giving a peak in $P(I)$ is analyzed by the quantum-mechanical model proposed by Fistul *et al*. [30]. In this model, $I_s(P_{\mathrm{MW}})$ is reproduced by using a quality factor Q of the resonance as a fitting parameter. Takahashi *et al*. obtained the following results. Below 10 K, where the MQT-like behavior was observed without the microwave irradiation, the measured $I_s(P_{\mathrm{MW}})$ was successfully reproduced by the numerical calculations assuming $Q \sim 5$. On the other hand, at higher temperatures above 16 K, the measured $I_s(P_{\mathrm{MW}})$ showed a partial agreement with the calculated $I_s(P_{\mathrm{MW}})$, by assuming the slightly decreasing $Q(\sim 3.7 - 3.2)$ with increasing temperature.

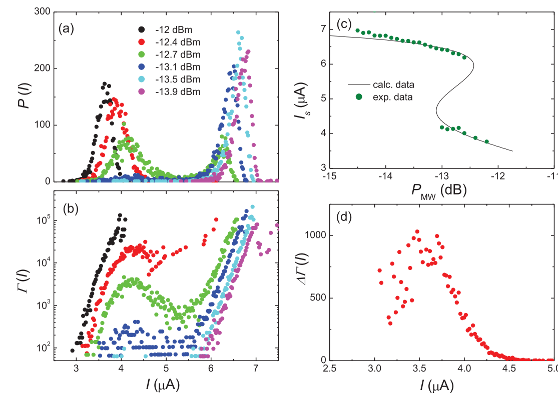

Fig. 6 Schematic examples of (a) the microwave-resonant escapes in $P(I)$, (b) the corresponding switching rate $\Gamma(I)$, (c) the most-frequently switching current $I_s(P_{\mathrm{MW}})$ with a numerical calculation curve and (d) the microwave enhancement of the switching rate.

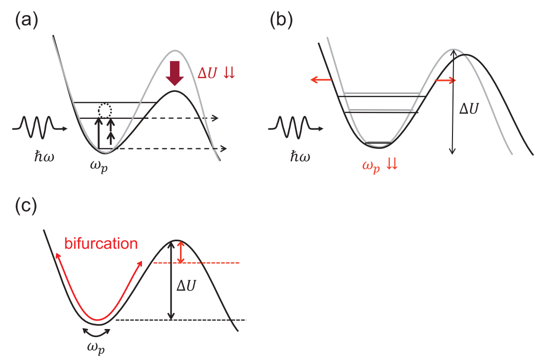

Fig. 7 (a) Washboard potential with the reduced tunnel barrier ΔU by irradiating microwaves. (b) Washboard potential with the expanding curvature around the bottom. (c) Bifurcation oscillation in the potential well.

In the proposed model by Fistul *et al*. [30], the strong microwave field effectively decreases the height of the potential barrier $\Delta U(I_s)$, as shown in Fig. 7 (a). This leads to an increase of quantum escapes. Kitano *et al*. [31] confirmed such a decrease of $\Delta U(I_s)$ by analyzing the measured results of Takahashi *et al*. [29]. Kitano *et al*. also obtained the microwave enhancement of the switching rate. It is given by $\Delta\Gamma(I)(\equiv [\Gamma(P_{\mathrm{MW}}, I) - \Gamma(0, I)]/\Gamma(0, I))$, as shown in Figs. 6 (b) and 6 (d). Here, $\Gamma(P_{\mathrm{MW}}, I)$ and $\Gamma(0, I)$ are the switching rate with and without the microwave irradiation, respectively. They observed a broad resonance peak in $\Delta\Gamma(I)$ below 10 K, which could be fitted to the standard Lorentzian curve with $Q \sim 2$, while no peak in $\Delta\Gamma(I)$ was obtained above 16 K.

The resonant enhancement of $\Delta\Gamma(I)$ by irradiating microwaves has been regarded as a consequence of the formation of ELQ [10], [13], [32]. Thus, the resonant peak observed in $\Delta\Gamma(I)$ seems to provide the strong evidence for the occurrence of the MQT and ELQ for the 2nd SWs. However, it remains unresolved that I_s is strongly reduced by the microwave irradiation and that the quality factor Q of the resonance is much smaller than that for the 1st SW in the MQT state (for instance, $Q > 30$ in Ref. [13]). Another important problem is that the crossover temperature to the MQT state is too high to be explained by the conventional theory based on the RCSJ model of the one-dimensional JJ array. This suggests the critical importance of the strong coupling between JJs, which is peculiar to IJJs. Thus, the effects of the inductive or capacitive coupling between JJs to the MQT rate were theoretically investigated ([33], [34]). Kawabata *et al*. showed that the strongly inductive coupling between JJs favored the collective MQT occurring along the diagonal direction of the two-dimensional washboard potential [33]. Next, Chizaki *et al*. calculated the MQT rate under the time-dependent periodic perturbation, describing the 2nd SWs where one of the capacitively coupled JJs was switched to the voltage state. They showed that $\Gamma(I)$ for the 2nd SWs was always larger than that for the 1st SWs and was resonantly enhanced at the specific bias current satisfying the resonance condition that $\omega = n\omega_p$ [34]. This condition represents that the population of the excited states with a lower value of ΔU is increased by the excitation induced by the periodic perturbation and that the quantum escapes from the excited states is resonantly increased.

Although both theoretical calculations suggested the enhancement of the MQT rate due to the inductive or capacitive coupling between JJs, the increase of T_{cr} to the MQT state was not expected at all. These results need another interpretation about the anomalous MQT-like behavior observed for the 2nd SWs and the higher order of SWs, as will be discussed in the next section.

3. Nonlinear Bifurcation Effects in the IJJs

A small distance between JJs and a small thickness of the superconducting layer, as shown in Figs. 1 (a) and 1 (b), are quite important to consider the phase dynamics of IJJs. The early numerical study, reported by Machida *et al*. [35], indicates that the charge imbalance between the neighboring JJs, derived from these features, provides a physical origin for both the multiply branched *I-V* curves and the Josephson plasma resonance (JPR) observed for the IJJs. In this study, it was also pointed out that the total driving force acting on each JJ depended on the dynamical state of neighboring JJs. Note that such a dynamical effect works between the JJs closely connected and that it is regarded as a kind of the interaction between JJs, which is quite different from the inductive or capacitive coupling between JJs.

Kitano *et al*. proposed that the dynamical state of the phase-switched junction could affect the phase escapes occurring at the neighboring junction, by analyzing the measured data of $\Gamma(I)$ for the higher order SWs [36]. They found that the multiple PR effects were more prominent for higher order SWs and that a similar increase of the PR effects was observed for $\Gamma(I)$ under the microwave irradiation. This suggests that the AC Josephson current generating in the phase switched junction plays an important role similar to the dynamical driving force given by the microwave irradiation.

In the case of the higher order SWs, it is considered that the dynamical driving force is generated during the phase running state, that is, between the neighboring order of SWs, as denoted by red arrows in Fig. 2 (a). This suggests a possibility that the anomalous behavior for the 2nd SWs does not appear in the IJJs with nearly identical switching current distribution for the 1st and 2nd SWs. By using such IJJs, Yamaguchi *et al*. confirmed that the bath temperature dependence of $T_{\rm eff}$ for the 2nd SWs became almost the same as that for the 1st SWs [37]. Similar results have also been reported for the mesa-type Bi2223-IJJs, where the 1st and 2nd switching current were nearly identical [38]. These results strongly suggest that the anomalous enhancement of T^* for the 2nd SWs originates from the phase running state after the 1st SW.

Yamaguchi *et al*. also showed that the microwave-induced resonant escapes under the strong microwave irradiation brought the strong decrease of ω_p rather than that of $\Delta U(I)$ for both the 1st and 2nd SWs [37]. In addition, they calculated the number of quantized energy levels formed in the potential well, by using the previous approach [39], and obtained the result that there were too many energy levels ($\gtrsim 20$ at $T = 1.7$ K for both SWs) to escape from the 1st excited state through the multi-photon absorption processes. These results show a sharp contrast to the previous studies ([29], [31]), where the resonant escapes were successfully explained by the effective reduction of $\Delta U(I)$ under the strong microwave irradiation, as proposed by Fistul *et al*. [30]. However, the two results discovered by Yamaguchi *et al*., that is, the decrease of ω_p with increasing the irradiation power and the deep potential well including many quantized energy levels, seem to provide an important key to settle the unresolved issue for higher order SWs.

Since the driving force expressed by a gradient of the washboard potential includes a nonlinearity of Josephson effects, the nonlinear dynamics is essentially important in the JJ system. In the conventional JJs, Yu *et al*. reported the observation of the classical resonant escapes due to a dynamical bifurcation under strong AC driving [40]. Manucharyan *et al*. pointed out that a JJ driven near a dynamical bifurcation point can amplify quantum signals [41]. The application to Josephson bifurcation amplifier has already been discussed for the readout of Josephson qubits [42]. Bifurcating nonlinear system is often described by using the so-called *Duffing oscillator* model where the nonlinearity is expressed by a cubic term of the driving force ([40], [41]). The increase of a periodic driving force such as the microwave field bends a resonance curve of the nonlinear oscillator towards lower frequencies. Thus, the resonant frequency is decreased with increasing the driving amplitude, as shown in Fig. 7 (b). Above a bifurcation point, the system switches from an oscillatory state with small amplitude to that with significantly large amplitude. Such a large amplitude can assist the phase escapes from a bottom in the deep potential well, by effectively decreasing the height of $\Delta U(I)$, as shown in Fig. 7 (c). It also gives a plausible explanation about the large reduction of I_s and the low value of Q at the microwave resonance. Therefore, the bifurcation scenario can explain almost all of the observed results for the microwave-induced resonant escapes for the higher order SWs.

In addition, the similarity between the microwave irradiation as the dynamical driving force and the AC Josephson current occurring in the phase running state after the phase escapes, pointed out by Kitano *et al.* [36], strongly suggests that the bifurcation scenario can also explain the anomalous MQT-like behavior observed for the higher order SWs without the microwave irradiation. In this case, the temperature-independent behavior of T_{eff} surviving up to T^* is derived from the characteristic energy of bifurcating oscillation rather than that of quantum fluctuation. Thus, the true MQT state is expected to emerge below the same crossover temperature as T_{cr} for the 1st SWs.

4. Toward the Implementation of IJJ-Based Superconducting Qubits

As shown in Fig. 5, it is expected that the MQT state realized in the IJJs is survived up to a few Kelvin, which is roughly an order of magnitude larger than the crossover temperature for the conventional JJs. Presently, Josephson qubits utilizing Al- or Nb based JJs are operated at the extremely low temperatures below 0.1 K and commercially available quantum computers prepare dilution refrigerator systems. If Josephson qubits based on IJJs are realized, the dilution refrigerator system is replaced by the ^{3}He refrigerator system. The operating frequency of qubits will be also shifted from the present microwave region to the submillimeter wave to THz wave region, according to the increase of the zero-bias Josephson plasma frequency ω_{p0}. Thus, the implementation of Josephson qubits utilizing IJJs strongly requires the development of THz electronics. Not only the emission and detection of the THz wave but also the fabrication of a compact cavity resonator operating at the THz frequencies are needed. The IJJ-based THz emitter is a very good candidate for this purpose. The bifurcation phenomenon expected to emerge in the phase running state of the IJJs is also available for the amplification of the readout signals.

Looking back into the history from the discovery of MQT to the successful implementation of charge qubits by Nakamura *et al.* [43], one can understand that the most important breakthrough is the *electromagnetic* isolation of small IJJs from bulk superconductor. Such isolation is crucially required to bring back the following uncertainty relation between phase and particle number in the Josephson system.

$$[\hat{\varphi}, \hat{N}] = i \quad (\leftrightarrow [\phi_0, 2e] = ih), \tag{3}$$

where i is an imaginary unit, ϕ_0 is the flux quantum and h is Planck's constant. Note that the uncertainty of φ and N in the usual JJs goes to zero and an infinity, respectively, as like as bulk superconductors. Therefore, the restitution of the uncertainty relation is a fundamental of the implementation of the *artificial atomic* state. Actually, in the case of charge qubits, a Cooper-pair box using the Cooper-pair blockade effect was prepared [43]. In flux and phase qubits, a SQUID loop and a spiral inductor with large inductance for a high impedance were prepared ([44]–[46]).

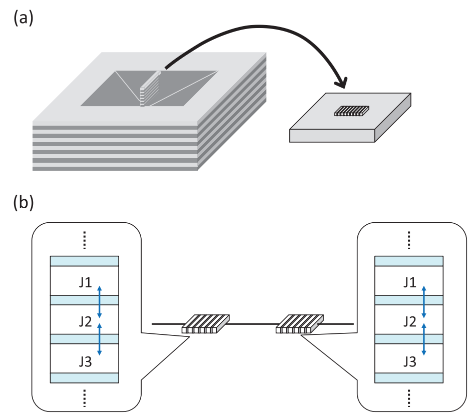

Fig. 8 Schematics of (a) a pick-up method of a small lamella by using FIB techniques, and (b) a rough idea that two superconducting qubits with internal degree of freedom, implemented in IJJs, quantum-mechanically comunicate with each other.

Unfortunately, the attempt to isolate the IJJ stack has not been reported yet. Note that both of the bridge- and mesa-type IJJs are continuously connected to a thick base crystal, as shown in Fig. 3. Thus, even if single IJJ was obtained between two slits in Fig. 3 (b), as previously reported by You *et al.* [47], the *electromagnetic* isolation of the IJJ is insufficient, in contrast to the present Josephson qubits. In addition, although the *stand-alone* mesa of IJJs, where the Bi2212 base crystal was cleaved away, has been prepared to improve the cooling performance of the THz wave emitter [48], it seems to be quite difficult to reproduce such an excellent exfoliation in much smaller sizes than the THz wave emitter.

Another idea for the isolated IJJs is the use of small lamella picked up from the base crystal by using FIB techniques, as shown in Fig. 8 (a). We have succeeded in obtaining the superconducting nanobridge with a cross sectional area of 0.06 $\mu\mathrm{m}^2$ by applying the FIB-based pickup method to iron chalcogenide superconductors [49]. The merit of this method is that the IJJs embedded into the small lamella lie on a substrate. This shows a sharp contrast to the situation of the bridge- and mesa-type IJJs, where the IJJs vertically stand on the substrate, as shown in Fig. 8 (a). The IJJs lying on the substrate favor the two-dimensional circuit fabrication, which is very matched with the modern trend of the superconducting circuits embedding Josephson qubits. On the other hand, the bridge- or mesa-type IJJs standing on the substrate seem to go together with the three-dimensional circuits, requiring a high level of implementation technique. Thus, the microfabrication of the picked up Bi2212 lamella is expected to be a breakthrough of the implementation of IJJ-based superconducting qubits.

As the first challenge for the isolated IJJs, our group tries to fabricate the IJJ device on the picked up Bi2212 lamella, by using the FIB techniques. Since the FIB etching process usually produces the amorphous layer on an etched surface, it is considered that the picked up Bi2212 lamella is covered by the amorphous layer. The recent study of transmission electron microscope (TEM) reported that the thickness of the amorphous layer was about 30 nm [50]. In contrast to iron chalcogenide superconductors, the amorphous layer of Bi2212 seems to be insulating or non-superconducting, since the crystallization treatment is often needed for the Bi22212 amorphous thin films [51]. Thus, one of the important problems to be resolved is a supply of electric power to the Bi2212 lamella. To supply DC current, the removal of the amorphous layer or the recrystallization of the lamella is needed, while the contact-free method is also expected to be necessary for the realization of the isolated IJJs, in the future.

Finally, I would like to refer to an interesting aspect of the IJJ-based superconducting qubits. Because of the strong interaction between neighboring JJs in the IJJs, it is almost impossible to mount a single qubit in each JJ of the IJJs and to coordinate with each other. Rather, the qubit state implemented in IJJs can possibly be treated as one with internal degree of freedom, which is similar to an atom with the fine structure splitting and hyperfine structure splitting, or the quantum-mechanically entangled states where the qubit states in several JJs of IJJs are entangled, similar to the previous proposal by Machida and Koyama [52]. Perhaps, this feature may be difficult to be mastered for the quantum information processing, but it is enough attractive as a possible new type of Josephson qubits. Figure 8 (b) shows a rough idea that two superconducting qubits with internal degree of freedom, implemented in IJJs, quantum-mechanically comunicate with each other. Although it is unclear whether this type of qubits are useful for a future quantum information processing, the IJJ-based superconducting qubit is obviously an interesting target which can provide novel applications of Josephson qubits.

5. Concluding Remarks

In this article, I described the elucidation of the phase switching dynamics in the IJJs where many JJs are closely connected with each other in an atomic scale. The experimental confirmation of MQT and ELQ occurring below $T_{\rm cr}$ for the 1st SWs is almost established. On the other hand, the controversial issues of the MQT-like behaviors and the microwave-induced resonant behaviors for the higher order SWs more than the 2nd SWs seem to be successfully explained by the nonlinear bifurcation emerging in the phase running state of the IJJs or under the microwave irradiation. Toward the implementation of the IJJ-based superconducting qubits, the isolation of the small IJJs is crucially important, as like the present Josephson qubits. The use of the picked-up lamella of Bi2212 single crystal is a good candidate for this purpose. Although there are many challenges to be resolved toward the realization of the IJJ-based superconducting qubits, the progress of the microfabrication techniques for single crystalline superconductors and the device working principle for qubits with internal degree of freedom or quantum entanglement will open up the availability of novel applications of the IJJ-based superconducting devices.

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Haruhisa Kitano received the Ph.D. degree from the University of Tokyo, Tokyo, Japan, in 1999. Presently, he is a professor in the Department of Physics, Aoyama Gakuin University. His current research interest is the fundamental study of nonequilibrium superconductivity occurring in small junctions and narrow microbridges made of cuprate and iron based superconductors.