INVITED PAPER   *Special Section on Recent Progress in Electromagnetic Theory and Its Application*

Theoretical Study on Wave Propagation and Scattering in Random Media and Its Application

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SUMMARY   The theoretical studies conducted mainly by the author are reviewed on (1) derivation of arbitrary order moment equations and solutions of some equations, (2) scattering by many particles and the effective medium constant of random medium, (3) scattering by a conducting body in random media and (4) spatially partially-coherent wave scattering, with application to satellite communications, artificial material development, and sensing and radar technology. The leading research results are described with many references; and also unsolved subjects in the above four studies are touched.
key words: *random medium, propagation, scattering, moment equation, effective medium constant, scattering cross-section, partial coherence*

1. Introduction

Theoretical study on wave propagation and scattering in random media has a long history and is said to originate in analysis of the twinkle of the stars. Multiple scattering in random media was investigated in the 1960s–1970s mainly in USA and USSR. The driving force behind the investigation was based on the solution of practical and core subjects for evaluating effects of random media on communications and remote sensing. The investigation is deeply related to physics and mathematics and has been carried out also from academic points of view [1]–[4]. Now the theoretical study has been worldwide promoted from different angles.

Deciding on the final goal that the theoretical results are used in various regions of science and technology and solving subjects in order to accomplish the goal, the author has been studying random medium problems from the beginning of 1970. The author's study corresponds to engineering including basis and application in Table 1, and has been directed to the development of creative methods and applications.

As a result of the study, the following four subjects are described in this paper: (1) derivation and analysis of moment equations, (2) scattering by many particles and the effective medium constant of random medium, (3) scattering by a conducting body in random media and (4) spatially partially-coherent wave scattering. The paper space is limited so that main points of research on the above subjects would be shortly described and detailed results and data should be obtained from references.

2. Derivation and Analysis of Moment Equations

Suppose that the dielectric constant of random medium ε is a random function of time and space, the time change is much slower in the medium than the wave velocity, and the ergodic hypothesis is valid. Then ε may be treated as a function of space; and Maxwell's equations for electromagnetic waves and Helmholtz's equation for scalar waves, respectively, are given as follows.

$$\begin{cases} \nabla \times \boldsymbol{E}(\boldsymbol{r},\,\omega) & = -\mathrm{j}\omega\mu_0\boldsymbol{H}(\boldsymbol{r},\,\omega) \\ \nabla \times \boldsymbol{H}(\boldsymbol{r},\,\omega) & = \mathrm{j}\omega\varepsilon(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r},\,\omega) \end{cases} \tag{1}$$

$$\left(\nabla^2 + k^2(\boldsymbol{r})\right)u(\boldsymbol{r},\,\omega) = 0 \tag{2}$$

where the wave number $k(\boldsymbol{r}) = \omega\sqrt{\varepsilon(\boldsymbol{r})\mu_0}$, the magnetic permeability $\mu = \mu_0$ (constant), and the time factor $\exp(\mathrm{j}\omega t)$ is suppressed.

It is difficult that we express solutions of (1) and (2) explicitly in a compact form by using known functions because $\varepsilon(\boldsymbol{r})$ is a random function. In case effects of the random medium are weak, we can apply approximate methods useful for analyzing waves in inhomogeneous media: e.g. geometrical optics approximation and Rytov approximation in asymptotic analysis, and Born approximation in perturbation analysis. According to these methods, the amplitude or phase of wave depends linearly on $\varepsilon(\boldsymbol{r})$, and the multiple scattering of wave by $\varepsilon(\boldsymbol{r})$ is not sufficiently considered. Therefore analytic methods applicable also to the case that effects of the random medium become strong were studied mainly in USA and USSR during times from 1960s to 1970s. As a result, the moments required in practical applications, which are statistics of electromagnetic waves: e.g. $\langle \boldsymbol{E}(1) \rangle$, $\langle \boldsymbol{E}(1)\boldsymbol{E}^*(2) \rangle$ and $\langle |\boldsymbol{E}(1)|^2|\boldsymbol{E}(2)|^2 \rangle$, have been analyzed, where the number in parentheses indicates the position in space and occasionally the frequency, and the asterisk denotes complex conjugate.

The above analysis paid attention to the moments is called as dishonest method [1] by which practically useful results have been obtained up to now. In general, any order moments are given as

$$M_{\mu\nu}(p, q) = \left\langle \prod_{m=0}^{\mu} E_{im}(p_m) \prod_{n=0}^{\nu} E_{jn}^*(q_n) \right\rangle \tag{3}$$

where E_{im}, E_{jn} are the vector components of $\boldsymbol{E}$, and p_m, q_n denote the space position and the frequency occasionally.

Table 1 A classification of studies on wave propagation in random media.

Mathematics	Engineering	
	Basis	Application
stochastic differential equations	methods for analyzing wave motion	communication systems
existence	moment equations	radars
uniqueness	characteristics of wave fields	remote sensing
stochastic properties of solutions	analytic expressions of moments	imaging
⋮	⋮	⋮

Moment equations that (3) satisfies were derived for successively forward-scattered waves on the assumption that $\varepsilon(\boldsymbol{r})$ is Gaussian random and the correlation length l is much larger than the wave length λ in free space: $kl \gg 1$, $k = 2\pi/\lambda$. The deviation was done from different approaches by assuming in addition that $\varepsilon(\boldsymbol{r})$ obeys Markov process in the wave-propagation direction, except for the author. A phase-changing screen model for random medium is physically intelligible to the layman [5]. However my approach based only on $kl \gg 1$ does not need the additional assumption and permits the inhomogeneous randomness of $\varepsilon(\boldsymbol{r})$ in the wave-propagation direction [6]–[9], which result is useful for practical applications such as satellite communications. Analyzing the moment equations, the author has presented the following results: the mechanism of spot dancing of wave beams [10], reciprocity of electromagnetic waves in random media [11], [12] and analytic expressions of second moments [13]–[16]. These results have been applied to an evaluation of antenna gains [17], restoration of images degraded by random media [18], [19] and the BER analysis in Ka band satellite communications [20]. Much more still remains to be investigated in practical application.

3. Scattering by Many Particles and the Effective Medium Constant of Random Medium

Consider the scalar wave scattering problem shown in Fig. 1 where $\varepsilon(\boldsymbol{r})$ in (2) is given by $\varepsilon(\boldsymbol{r}) = \varepsilon_0\big(1 + \Delta\varepsilon(\boldsymbol{r})\big)$ and $\Delta\varepsilon(\boldsymbol{r})$ indicates the distribution of relative dielectric constants of particles. If we assume the wave in V as the incident wave: i.e. Born approximation, then the assumption corresponds to the first term approximation $T(\boldsymbol{r}'', \boldsymbol{r}') = k^2\Delta\varepsilon(\boldsymbol{r}'')\delta(\boldsymbol{r}'' - \boldsymbol{r}')$ in Fig. 1. The solution of this problem is to obtain $T(\boldsymbol{r}'', \boldsymbol{r}')$; however, it is not easy for any distribution of $\Delta\varepsilon(\boldsymbol{r})$.

Scattering by many particles has been analyzed by different methods according as the distribution of particles is periodic or random. To the periodic case Floquet's theorem applies and the analysis results in scattering by only one particle; on the other hand, to the random case, so-called multiple scattering methods have been applied. Although periodic and random distributions are quite distinct from each other, the discrimination of the distributions becomes difficult when using waves of much longer wavelength than the average distance between particles. The author presented a method, called DUR method, applicable to the transition case from periodic to random distribution for scalar waves [21] and electromagnetic waves [22]. Figure 2 shows con-
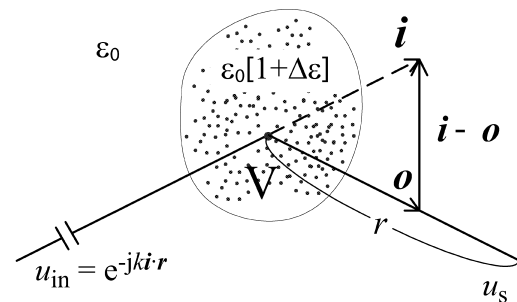
The scattered far-wave for an incident plane-wave:

$$u_{\rm s}(\boldsymbol{r}) = \frac{e^{-\mathrm{j}kr}}{r}\left[\frac{1}{4\pi}\hat{T}(\boldsymbol{0}, \boldsymbol{i})\right]$$

$$\hat{T}(\boldsymbol{0}, \boldsymbol{i}) = \int_V d\boldsymbol{r}' \int_V d\boldsymbol{r}'' e^{\mathrm{j}\boldsymbol{k}_0 \cdot \boldsymbol{r}''} T(\boldsymbol{r}'', \boldsymbol{r}') e^{-\mathrm{j}\boldsymbol{k}_i \cdot \boldsymbol{r}'}$$

in which, using Green's function in free space $G(\boldsymbol{r}|\boldsymbol{r}')$,

$$T(\boldsymbol{r}'', \boldsymbol{r}') = k^2\Delta\varepsilon(\boldsymbol{r}'')\delta(\boldsymbol{r}'' - \boldsymbol{r}')$$
$$+ \ k^2\Delta\varepsilon(\boldsymbol{r}'')\int_V G(\boldsymbol{r}''|\boldsymbol{r}_1)T(\boldsymbol{r}_1, \boldsymbol{r}')d\boldsymbol{r}_1$$

Fig. 1 Scattering by many particles.

ventional and my approaches where the particle distributions are described. Using DUR method equations were approximately derived for the coherent wave $\langle\boldsymbol{E}(\boldsymbol{r})\rangle$ and the second moment $\langle\boldsymbol{E}(\boldsymbol{r}_1)\boldsymbol{E}^*(\boldsymbol{r}_2)\rangle$, and expressed in terms of two parameters of periodicity and randomness in the particle distribution.

By separating coherent Green's function $\langle G(\boldsymbol{r})\rangle$ into the periodicity and randomness parts where well-known periodic Green's function remains for non-random case and the periodicity in $\langle G(\boldsymbol{r})\rangle$ vanishes for strongly random case, the author investigated the condition for the distribution of particles to be random, which means that the periodicity part can be neglected in $\langle G(\boldsymbol{r})\rangle$. As a result, the conditions for dielectric spheres and cylinders were given by using the radius and relative dielectric constant of particles, the average distance between particles, and the frequency [23], [24].

According as the distribution of particles, effective dielectric constants of media $\langle\varepsilon(\boldsymbol{r})\rangle$ are depicted as Fig. 3. The author evaluated the $\langle\varepsilon(\boldsymbol{r})\rangle$ of media composed of dielectric spheres for the weakly and strongly random distributions: (b) and (c) in Fig. 3 and also did the $\langle\varepsilon(\boldsymbol{r})\rangle$ for the case of dielectric cylinders [25], [26]. The $\langle\varepsilon(\boldsymbol{r})\rangle$ of random media

has been evaluated by multiple scattering methods such as EFA: effective field approximation, QCA: quasi crystalline approximation and QCA-CP: QCA with Coherent Potential. As a result, the evaluation by DUR method is the best up to now and is applicable to random media composed of highly dielectric particles such as water drops for microwaves. This prominent result was applied to the detection problem of the water content in soil [27], [28] and also to the estimation of the effective medium parameters of random media composed of chiral particles [29]. The reference [30] shows the difference in multiple scattering among EFA, QCA, QCA-CP and DUR method. It is not easy to analyze the effective medium parameters if random medium consists of several kinds of particles. Therefore the analysis has been done numerically. Because the numerical analysis causes a large-scale calculation problem, a fast computation method adequate to support the time, memory and accuracy has been required; accordingly, we have studied the method and obtained some useful results [31], [32]. The study still continues on scattering by many particles.
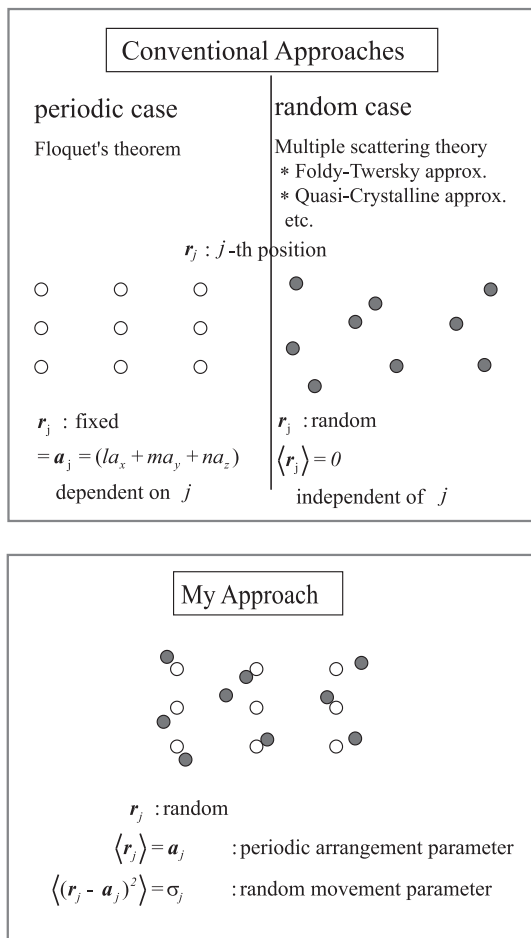
4. Scattering by a Conducting Body in Random Media

To develop the radar technology, scattering by a target in random media should be analyzed as a boundary value problem. The author proposed a method to do that to the scattering problem shown in Fig. 4 [33], [34]. The outline of the method is given in Fig. 5, where the case of random medium is compared with the case of free space and the current generator plays an important role for the formulation. The generator, whose name is unfamiliar, depends only on the body, is valid for any incident wave and can be obtained by Yasuura's method.

Using the proposed method, average scattering cross-sections have been calculated and shown to have characteristics peculiar to the case of random medium [35]–[39]. However, the vector analysis of the three dimensional problems remains untouched because of complexity. Through
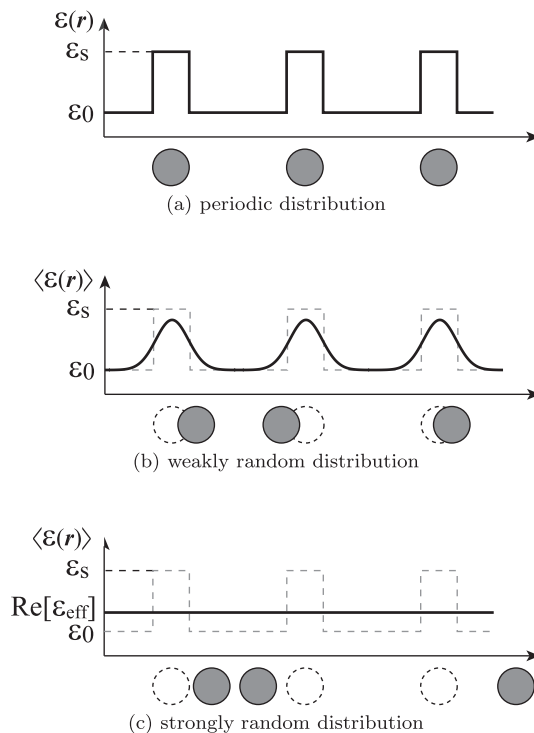
Fig. 3 Effective dielectric constants of media for (a) periodic, (b) weakly random and (c) strongly random distributions of particles.
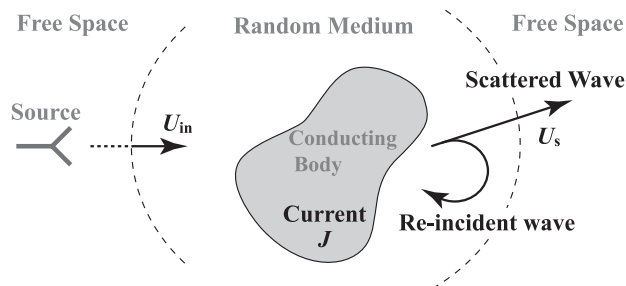
Fig. 2 Comparison of particle distributions: (a) Conventional distributions and (b) Proposed distribution.

Fig. 4 Scattering by a conducting body in random media.

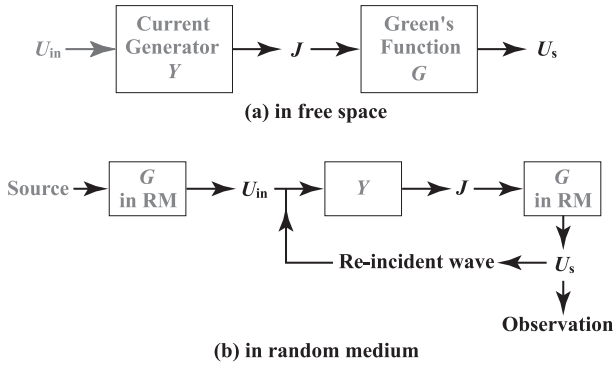**(a) in free space**

(b) in random medium

Fig. 5 A schematic diagram for solving the scattering problem where U_{in} is incident wave, J current, U_s scattered wave, and RM indicates random medium.
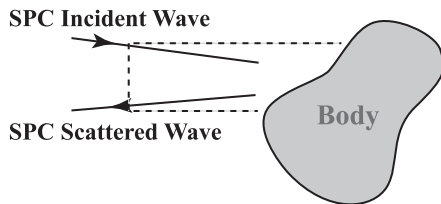

issue 1: scattering for the incidence of spatially partially-coherent (SPC) wave

issue 2: statistical coupling of incident and scattered SPC waves through random medium
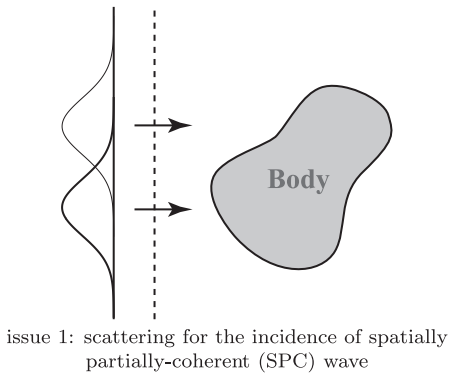
Fig. 6 Two issues in the scattering problem.

the calculation it became clear [40], [41] that this scattering problem consists of two physical issues: that is, scattering by spatially partially-coherent (SPC) incident waves and statistical coupling of incident and scattered SPC waves through random medium, as shown in Fig. 6. Well-known backscattering enhancement occurs by issue 2 in Fig. 6; however, in practice such as radar engineering, we need the quantitative estimate of scattering cross-sections by taking account of issue 1. This fact is closely related to the subject of the next chapter.

5. Spatially Partially-Coherent Wave Scattering

Scattering by a body in free space has been basically studied on the assumption that the incident wave is perfectly-coherent in space. On the basic assumption many research results have been obtained and applied to practical
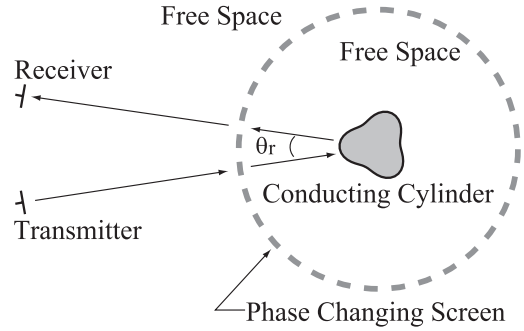

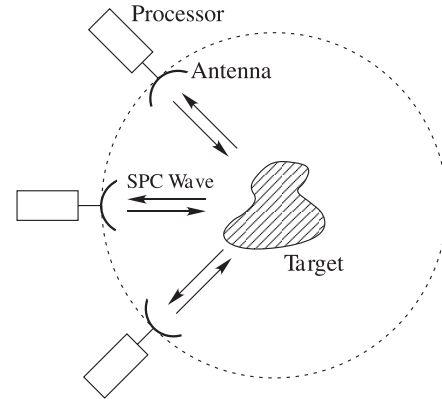**Fig. 7** Scattering from a body surrounded by a phase changing screen.

Fig. 8 A prototype of circular SAR.

cases; currently, the study of the scattering continues using large-scale computation techniques. If the incident wave is partially-coherent in space: i.e. SPC wave, then there is some possibility that the scattering cross-section of a body in free space differs much from that for the incidence of perfectly-coherent wave because large effects of the issue 1 described in Sect. 4 are partly known to the case of random medium. Therefore we need to investigate the scattering for the SPC wave incidence as a new subject. In order to do it, we consider the scattering problem shown in Fig. 7 where a conducting body is surrounded by a phase changing screen with an infinitesimal shin layer. When changing randomly the phase, we can obtain the scattering cross-section similar to that for the case of random medium shown in Fig. 4 [42], [43]. This study is connected with the development of a new SAR system using SPC waves, as shown in Fig. 8. Scattering from a body illuminated by SPC waves is a challenging subject.

6. Concluding Remarks

The theoretical studies conducted mainly by the author are reviewed on wave propagation and scattering in random media and its application: in particular, on (1) derivation of arbitrary order moment equations and solutions of some equations, (2) scattering by many particles and the effective medium constant of random medium, (3) scattering by a conducting body in random media and (4) spatially

partially-coherent wave scattering, with application to satellite communications, artificial material development, and sensing and radar technology. The leading and/or outstanding points of the above researches are merely described and detailed results and data are referred to representative articles already published.

Acknowledgements

References

[1] J.B. Keller, "Wave propagation in random media," Proc. Symp. App. Math., 13., pp.227–246, Amer. Math. Soc., Providence, R.I., 1962.

[2] U. Frish, "Wave propagation in random media," in Probabilistic methods in applied mathematics, ed. A.T. Bharucha-Reid, vol.1, pp.76–198, Academic Press, New York, 1968.

[3] V.I. Tatarski, The effects of the turbulent atmosphere on wave propagation, Israel Program for Scientific Translations, Jerusalem, 1971.

[4] H. Bremmer, "General remarks concerning theories dealing with scattering and diffraction in random media," Radio Sci., vol.29, no.6, pp.511–534, 1973.

[5] B.J. Uscinski, Elements of Wave Propagation in Random Media, McGraw-Hill, 1977.

[6] M. Tateiba, "Successively forward-scattered wave propagating through a random medium," IEICE Trans. Commun. (Japanese Edition), vol.56-B, no.1, pp.1–8, Jan. 1973.

[7] M. Tateiba, "On the equations for waves propagating through random media," Tech. Meeting on EM Theory, IEEJ (in Japanese), EMT-73, 1, 1973.

[8] M. Tateiba, "Moment equation of a wave propagating through random media," Mem. Fac. Eng. Kyushu Univ., vol.33, no.4, pp.129–137, 1974.

[9] M. Tateiba, "Validity of a selective summation technique in analysis of wave propagation through random media," Radio Sci., vol.17, no.2, pp.339–347, 1982 and Proc. URSI Int. Symp. on EM Waves, 1980.

[10] M. Tateiba, "Mechanism of spot dancing," IEEE Trans. Antennas Propag., vol.AP-23, no.4, pp.493–497, 1975.

[11] M. Tateiba, "Numerical analysis of nonreciprocity for spatial coherence and spot dancing in random media," Radio Sci., vol.17, no.6, pp.1531–1535, 1982.

[12] M. Tateiba, "The Lorentz reciprocity in random media — The derivation from Maxwell's equations," Radio Sci., vol.26, no.2, pp.499–503, 1991 and Proc.URSI Int. Symp. on EM Waves, 1989.

[13] M. Tateiba, "Multiple scattering analysis of optical wave propagation through inhomogeneous random media," Radio Sci., vol.17, no.1, pp.205–210, 1982.

[14] M. Tateiba, "An exact solution of the moment equation in a special case and the log-normal distribution," IECE Trans., vol.E66, no.4, pp.223–228, April 1983.

[15] M. Tateiba, "Some useful expressions for coherence functions propagated through random media," Radio Sci., vol.20, no.5, pp.1019–1024, 1985 and Proc. URSI Int. Symp. on EM Waves, 1983.

[16] M. Tateiba and Y. Iseki, "An analytic solution of the two-frequency moment equation in an inhomogeneous turbulent medium," Proc. Int. Symp. on Antennas and Propagation, 1989.

[17] M. Tateiba, "Analysis of antenna gain reduction due to ionospheric and atmospheric turbulence," Proc. Int. Symp. on Antennas and Propagation, 1985.

[18] M. Tateiba and K. Fujisaki, "Analysis of the intensity fluctuation of reconstructed images due to the atmospheric and ionospheric turbulence in satellite holographic imaging," IEICE Trans., vol.E73, no.4, pp.459–461, April 1990.

[19] K. Fujisaki and M. Tateiba, "Restoration of holographic image degraded by atmospheric turbulence through spatial filtering," J. Electromagnetic Waves and Applications, vol.8, no.3, pp.315–327, 1994.

[20] T. Hanada, K. Fujisaki, and M. Tateiba, "Theoretical analysis of bit error rate of satellite communication in Ka-Band under spot dancing and decrease in spatial coherence caused by atmospheric turbulence," Progress In Electromagnetics Research C, vol.3, pp.225–245, 2008.

[21] M. Tateiba, "A new approach to the problem of wave scattering by many particles," Radio Sci., vol.22, no.6, pp.881–884, 1987 and Proc. URSI Int. Symp. on EM Waves, 1986.

[22] M. Tateiba, "Electromagnetic wave scattering in media whose particles are randomly displaced from a uniformly ordered spatial distribution (Invited)," IEICE Trans. Electron., vol.E78-C, no.10, pp.1357–1365, Oct. 1995.

[23] M. Tateiba and Y. Nanbu, "The condition for the distribution of dielectric cylinders to be random — The derivation from the analysis of coherent fields," IEICE Trans., vol.E74, no.5, pp.1055–1058, May 1991.

[24] M. Tateiba and Y. Nanbu, "The condition for the distribution of many dielectric spheres to be random for a coherent field," Radio Sci., vol.28, no.6, pp.1203–1210, 1993 and M. Tateiba, Y. Nanbu and K. Takayama, Proc. URSI Int. Symp. on EM Theory, 1992.

[25] Y. Nanbu and M. Tateiba, "A comparative study of the effective dielectric constant of a medium containing randomly distributed dielectric spheres embedded in a homogeneous background medium," Waves in Random Media, vol.6, no.4, pp.347–360, 1996.

[26] Y. Nanbu and M. Tateiba, "Polarization effects on the effective dielectric constant of a medium containing randomly distributed dielectric cylinders," IEICE Trans. Electron., vol.E79-C, no.10, pp.1334–1337, Oct. 1996.

[27] T. Matsuoka and M. Tateiba, "Comparison of scattered power from a layer with randomly distributed lossy spheres of high dielectric constant by using radiative transfer theory," IEICE Trans. Electron., vol.E83-C, no.12, pp.1803–1808, Dec. 2000.

[28] T. Matsuoka and M. Tateiba, "Analysis of wave scattering from three layers of random medium by using a radiative transfer equation," IEEJ Trans. FM (Japanese Edition), vol.123, no.12, pp.1193–1198, 2003.

[29] Y. Nanbu, T. Matsuoka, and M. Tateiba, "The effective constitutive parameters of a random medium containing small chiral spheres," IEEJ Trans. FM (Japanese Edition), vol.123-A, no.3, pp.259–264, 2003.

[30] M. Tateiba and T. Matsuoka, "Electromagnetic wave scattering by many particles and its applications," Electron. and Commun. in Japan, Part II: Electronics, vol.88, no.10, pp.10–18, 2005.

[31] N. Nakashima and M. Tateiba, "Fast computation for electromagnetic wave scattering problem with many dielectric cylinder," IPSJ Trans. Compu. System, vol.44, no.SIG6, pp.18–26, 2003.

[32] N. Nakashima and M. Tateiba, "Greengard-rokhlin's fast multipole algorithm for numerical calculation of scattering by N conducting circular cylinders," IEICE Trans. Electron., vol.E88-C, no.7, pp.2158–2166, July 2005.

[33] M. Tateiba and E. Tomita, "Theory of scalar wave scattering from a conducting target in random media," IEICE Trans. Electron., vol.E75-C, no.1, pp.101–106, Jan. 1992.

[34] M. Tateiba and Z.Q. Meng, "Wave scattering from conducting bodies embedded in random media — Theory and numerical results," PIER 14, pp.317–361, EMW Pub., Cambridge, Massachusetts, 1996.

[35] Z.Q. Meng and M. Tateiba, "Radar cross-sections of conducting elliptic cylinders embedded in strong continuous random media,"

Waves in Random Media, vol.6, no.4, pp.335–345, 1996.

[36] M. Tateiba and Z.Q. Meng, "Radar cross-sections of conducting targets surrounded by random media (invited)," IEICE Trans. Electron. (Japanese Edition), vol.J84-C, no.11, pp.1031–1039, Nov. 2001.

[37] H. El-Ocla and M. Tateiba, "Strong backscattering enhancement for partially convex targets in random media," Waves in Random Media, vol.11, no.1, pp.21–32, 2001.

[38] H. El-Ocla and M. Tateiba, "Backscattering enhancement for partially convex targets of large sizes in continuous random media for E-wave incidence," Waves in Random Media, vol.12, no.3, pp.387–397, 2002.

[39] M. Tateiba, Z.Q. Meng, and H. El-Ocla, "Scattering by conducting bodies in random media (invited)," IEEJ Trans. FM, vol.124, no.12, pp.1094–1100, 2004.

[40] M. Tateiba, Z.Q. Meng, and H. El-Ocla, "Spatially partially-coherent wave scattering by a body in a random medium," Proc. PIERS, p.236, 2003.

[41] M. Tateiba, "Electromagnetic wave sensing in random media (plenary)," Proc. Sino-Japan Joint Meeting on OFSET, pp.9–14, 2004.

[42] K. Fujisaki, K. Haruta, Z.Q. Meng, and M. Tateiba, "The radar cross-section of a cylinder surrounded by a phase changing screen," Proc. PIERS, Hangzhou (CD-ROM), 2008.

[43] Z.Q. Meng, M. Tateiba, and K. Haruta, "Scattering from conducting cylinder surrounded by a phase-changing screen," Proc. APEMT, 2008.

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