LETTER

Two Classes of Optimal Ternary Cyclic Codes with Minimum Distance Four*

Chao HE[†a)], **Xiaoqiong RAN**[††b)], ***Nonmembers*****, *and* Rong LUO**[††c)], ***Member***

SUMMARY Cyclic codes are a subclass of linear codes and have applications in consumer electronics, data storage systems, and communication systems as they have efficient encoding and decoding algorithms. Let $C_{(t,e)}$ denote the cyclic code with two nonzero α^t and α^e, where α is a generator of $\mathbb{F}_{3^m}^*$. In this letter, we investigate the ternary cyclic codes with parameters $[3^m-1, 3^m-1-2m, 4]$ based on some results proposed by Ding and Helleseth in 2013. Two new classes of optimal ternary cyclic codes $C_{(t,e)}$ are presented by choosing the proper t and e and determining the solutions of certain equations over $\mathbb{F}_{3^m}$.

key words: *linear codes, ternary cyclic codes, optimal codes, sphere packing bound*

1. Introduction

Let p be a prime. An $[n,k,d]$ linear code C over the finite field $\mathbb{F}_p$ is a k-dimensional subspace of $\mathbb{F}_p^n$ with minimum (Hamming) distance d, and is called cyclic if any cyclic shift of a codeword is another codeword of C. By identifying $(c_0, c_1, \cdots, c_{n-1}) \in C$ with

$$c_0 + c_1x + c_2x^2 + \cdots + c_{n-1}x^{n-1} \in \mathbb{F}_p[x]/(x^n-1),$$

any cyclic code C of length n over $\mathbb{F}_p$ corresponds to an ideal of polynomial residue class ring $\mathbb{F}_p[x]/(x^n-1)$. Note that every ideal of $\mathbb{F}_p[x]/(x^n-1)$ is principal. Thus, any cyclic code C can be expressed as $C = \langle g(x) \rangle$, where $g(x)$ is a monic polynomial with the least degree. The polynomial $g(x)$ is called the generator polynomial of C.

Cyclic codes are a subclass of linear codes and have important applications in consumer electronics, data storage systems, and communication systems as they have efficient encoding and decoding algorithms compared with the linear block codes [3]. They also have applications in cryptography [4], [5] and sequence design [6]. Let α be a generator of $\mathbb{F}_{3^m}^* = \mathbb{F}_{3^m} \setminus \{0\}$, $m_{\alpha^t}(x)$ denote the minimal polynomial of α^t over $\mathbb{F}_3$, and $C_{(t,e)}$ be the class of cyclic codes over $\mathbb{F}_3$ with the generator $m_{\alpha^t}(x)$ and $m_{\alpha^e}(x)$ where $1 \le e, t \le 3^m-1$. Ding and Helleseth [7] constructed several classes of optimal ternary cyclic codes $C_{(1,e)}$ with parameters $[3^m-1, 3^m-1-2m, 4]$ by using almost perfect nonlinear monomials and some other monomials over F_{3^m}. Subsequently, many classes of optimal ternary cyclic codes $C_{(1,e)}$ with parameters $[3^m-1, 3^m-1-2m, 4]$ were constructed successively [8]–[22].

In this letter, we will present two new classes of optimal ternary cyclic codes with parameters $[3^m-1, 3^m-1-2m, 4]$ by analyzing the solutions of certain equations over F_{3^m}. It will be shown that our results about the two classes of optimal ternary cyclic codes are extensions of some previous results [15].

This letter is organized as follows. Some useful lemmas are given in Sect. 2. In Sect. 3, we presents an effective and fast method to determine whether ternary cyclic codes $C_{(t,e)}$ with exponents $(t,e) = (\frac{3^m+1}{2}, 3^h + 2\cdot 3^i)$ are optimal. Moreover, we show that ternary cyclic codes $C_{(t,e)}$ are optimal for $i = 0,1,2$. Section 4 concludes the letter.

2. Preliminaries

The following lemmas will be frequently used throughout the letter.

Lemma 1. ([7]) *For any integer* $1 \le e \le 3^m-2$. *Let* C_e *be the* 3*-cyclotomic coset module* 3^m-1 *containing* e. *The length of* C_e *is equal to* m *if* $\gcd(e, 3^m-1) = 2$.

Lemma 2. ([16]) *Let* t *be even and* e *be odd with* $\gcd(t, e, 3^m-1) = 1$. *Then the minimum distance of the cyclic code* $C_{(t,e)}$ *is no less than* 3.

Lemma 3. ([1]) *An irreducible polynomial over* $\mathbb{F}_{p^m}$ *of degree* n *remains irreducible over* $\mathbb{F}_{p^{ml}}$ *if and only if* $\gcd(n, l) = 1$.

Lemma 4. *Let* m *be an odd integer and* $\gcd(m,3) = 1$. *If* h *is an integer such that* $2h \equiv 1 \pmod m$, *then the equation*

$$\xi^{3^i}(\xi^{3^i} - \xi^{3^h}) = 1 \tag{1}$$

about ξ *has no solution in* $\mathbb{F}_{3^m}$ *for* $i = 0,1,2$.

Proof. Noting that $\xi = \pm 1$ and $\xi = 0$ are not solutions of (1). Suppose that $\xi \in \mathbb{F}_{3^m} \setminus \mathbb{F}_3$ is the solution of (1). Then

$$\xi^{3^h} = \xi^{3^i} - \frac{1}{\xi^{3^i}}. \tag{2}$$

Manuscript received June 25, 2023.
Manuscript revised September 25, 2023.
Manuscript publicized October 16, 2023.

[†]The author is with School of Teacher Education, Sichuan Vocational and Technical College, Suining 629000, China.
[††]The authors are with School of Mathematics, Southwest Jiaotong University, Chengdu 610031, China.

*This research was supported by the Foundation of Sichuan Vocational and Technical College under Grant 2022YZBO12 and the National Natural Science Foundation of China under Grant 12371524.

a) E-mail: chaohe@my.swjtu.edu.cn
b) E-mail: Ran2021@my.swjtu.edu.cn
c) E-mail: luorong@swjtu.edu.cn (Corresponding author)

DOI: 10.1587/transfun.2023EAL2055

Taking 3^h powers on both sides of (2), we have

$$\xi^{3^{2h}} = \left(\xi^{3^h} - \frac{1}{\xi^{3^h}}\right)^{3^i}. \quad (3)$$

Plugging (2) into (3), we have

$$\xi^{3^{2h}} = \left(\frac{\xi^4+1}{\xi^3-\xi}\right)^{3^{2i}}.$$

Noting that $\xi^{3^{2h}} = \xi^3$, since $2h \equiv 1 \pmod{m}$. Then we have

$$\xi^3(\xi^3-\xi)^{3^{2i}} - (\xi^4+1)^{3^{2i}} = 0. \quad (4)$$

If $i = 0$, we have $\xi^6+\xi^4-1 = (\xi^3+\xi^2+\xi+2)(\xi^3+2\xi^2+\xi+1) = 0$, where $\xi^3+\xi^2+\xi+2$ and $\xi^3+2\xi^2+\xi+1$ are irreducible over $\mathbb{F}_3$. For $\gcd(m,3) = 1$, according to Lemma 3, we know that the equation $\xi^6+\xi^4-1 = 0$ has no solution in $\mathbb{F}_{3^m}$. Therefore, in this case, we conclude that (1) has no solution in $\mathbb{F}_{3^m}$.

If $i = 1$, we have

$$((\xi^6-\xi^2+1)(\xi^6-\xi^4+\xi^2+1))^3 = 0, \quad (5)$$

where $\xi^6-\xi^2+1$ and $\xi^6-\xi^4+\xi^2+1$ are irreducible over $\mathbb{F}_3$. Nothing that $\gcd(m,6) = 1$, since m is odd and $\gcd(m,3) = 1$. By Lemma 3, $\xi^6-\xi^2+1$ and $\xi^6-\xi^4+\xi^2+1$ remain irreducible over $\mathbb{F}_3^m$. Therefore (5) has no solution in $\mathbb{F}_{3^m}$. Therefore,in this case, we conclude that (1) has no solution in $\mathbb{F}_{3^m}$.

If $i = 2$, we have

$$\xi^3(\xi^3-\xi)^{3^4} - (\xi^4+1)^{3^4} = 0. \quad (6)$$

By Magma computation, the Eq. (6) can be decomposed into the product of the irreducible factors as $(\xi^{54}+\xi^{50}+\xi^{46}+\xi^{44}-\xi^{40}+\xi^{38}+\xi^{26}-\xi^{22}+\xi^{20}+\xi^{18}+\xi^{14}-\xi^6+\xi^4+1)^3(\xi^{54}-\xi^{50}-\xi^{44}+\xi^{42}+\xi^{38}-\xi^{32}+\xi^{30}+\xi^{28}+\xi^{24}+\xi^{22}-\xi^{20}+\xi^{18}+\xi^{16}-\xi^{14}-\xi^{10}+\xi^8+\xi^6-\xi^4+1)^3 = 0$. Since m is odd and $\gcd(3,m) = 1$, $\gcd(54,m) = 1$. By Lemma 3, we have that (6) has no solution in $\mathbb{F}_{3^m}$. Therefore,in this case,we conclude that (1) has no solution in $\mathbb{F}_{3^m}$. □

Lemma 5. *Let h be an positive integer and $m = 2h-1$. Then*

1) $\gcd(3^h+2, 3^m-1) = 1$*;*
2) $\gcd(3^{h-1}+2, 3^m-1) = 1$ *if* $h \not\equiv 3 \pmod{5}$*;*
3) $\gcd(3^{h-2}+2, 3^m-1) = 1$ *if* $h \not\equiv 27 \pmod{53}$.

Proof. 1) By $3(3^m-1)-(3^h-2)(3^h+2) = 1$, we have $\gcd(3^h+2, 3^m-1) = 1$.

2) By $(3^m-1)-(3^h-6)(3^{h-1}+2) = 11$, we have $\gcd(3^{h-1}+2, 3^m-1) = \gcd(3^{h-1}+2, 11)$. Let $h-1 = 5k+j$ with k is some integer and $j \in \{0,1,3,4\}$. Then $\gcd(3^{h-1}+2, 11) = \gcd(3^{5k+j}+2, 11) = \gcd(3^j+2, 11) = 1$. Hence, $\gcd(3^{h-1}+2, 3^m-1) = 1$ if $h \not\equiv 3 \pmod{5}$.

3) By $(3^m-1)-(3^{h+1}-2\cdot 3^3)(3^{h-2}+2) = 107$, then $\gcd(3^{h-2}+2, 3^m-1) = \gcd(3^{h-2}+2, 107)$. It can be verified that $3^{53} \equiv 1 \pmod{107}$. Let $h-2 = 53k+j$ with k is some integer and $j \in \{0,\cdots,24\} \cup \{26,\cdots,52\}$. We have $\gcd(3^{h-2}+2, 107) = \gcd(3^{53k+j}+2, 107) = \gcd(3^j+2, 107)$. Hence, $\gcd(3^{h-2}+2, 3^m-1) = \gcd(3^j+2, 107) = 1$ if $h \not\equiv 27 \pmod{53}$. □

3. Optimal Ternary Codes with Minimum Distance 4

In this section, we consider the ternary cyclic code $C_{(t,e)}$, whose exponents t, e is in the form of

$$t = \frac{3^m+1}{2}, e = 3^h + 2\cdot 3^i \quad (7)$$

where $0 \le h < m-1, 0 \le i < h$ is integers and m is odd.

Theorem 1. *Let $m > 1$ be odd,* $\gcd(h-i, m) = 1$ *and t, e satisfying that (7). Then the ternary cyclic code $C_{(t,e)}$ has parameters $[3^m-1, 3^m-2m-1, 4]$ and is optimal if* $\gcd(3^{h-i}+2, 3^m-1) = 1$ *and the equation*

$$\lambda^{3^i}(\lambda^{3^i} - \lambda^{3^h}) = 1 \quad (8)$$

about λ has no solution in $\mathbb{F}_{3^m}$.

Proof. Note that $\gcd(t, 3^m-1) = 2$ since m is odd. Thus, by Lemma 1, we can obtain $|C_t| = m$. Similarly, $|C_e| = m$ since $\gcd(3^h+2\cdot 3^i, 3^m-1) = \gcd(3^{h-i}+2, 3^m-1) = 1$. Then, it can be readily verified that $C_t \cap C_e = \emptyset$. This implies that the dimension of $C_{(t,e)}$ is equal to 3^m-2m-1.

We now prove that the minimum distance d is equal to 4. To this end, we need to show that $C_{(t,e)}$ has no codewords of Hamming weight $\omega \in \{1,2,3\}$. The cyclic code $C_{(t,e)}$ has a codeword of Hamming weight ω if and only if there exist $c_1, c_2, \cdots, c_\omega \in \mathbb{F}_3^*$ and ω distinct elements $x_1, x_2, \cdots, x_\omega \in \mathbb{F}_{3^m}^*$ such that

$$\begin{cases} c_1x_1^t + c_2x_2^t + \cdots + c_\omega x_\omega^t = 0, \\ c_1x_1^e + c_2x_2^e + \cdots + c_\omega x_\omega^e = 0. \end{cases} \quad (9)$$

Since m is odd, we have $\gcd(\frac{3^m+1}{2}, 3^m-1) = 2$. Then we can get $\gcd(t, e, 3^m-1) = \gcd(\frac{3^m+1}{2}, 3^h+2\cdot 3^i, 3^m-1) = \gcd(2, 3^h+2\cdot 3^i) = 1$. By Lemma 2, we deduce that the minimum distance d of the cyclic code $C_{(t,e)}$ is no less than 3.

Next,we will show that $\omega \ne 3$. The cyclic code $C_{(t,e)}$ has no codewords of Hamming weight $\omega = 3$ if and only if (9) has no solution over $\mathbb{F}_{3^m}$ for $\omega = 3$. Let $x = \frac{x_1}{x_3}$ and $y = \frac{x_2}{x_3}$, we have $x, y \ne 0, 1$ and $x \ne y$. The Eq. (9) becomes

$$\begin{cases} c_1x^t + c_2y^t + c_3 = 0, \\ c_1x^e + c_2y^e + c_3 = 0. \end{cases}$$

Due to symmetry it is sufficient to consider the following two cases.

Case A: $c_1 = c_2 = c_3 = 1$. In this case, we have

$$\begin{cases} x^t + y^t + 1 = 0, \\ x^e + y^e + 1 = 0. \end{cases} \quad (10)$$

Recall that $t = \frac{3^m+1}{2} = \frac{3^m-1}{2} + 1$. We have $\alpha^t = \alpha$ if α is a square in $\mathbb{F}_{3^m}^*$ and otherwise $\alpha^t = -\alpha$. We distinguish among the following four subcases to prove that (10) cannot hold for x, y $(x \neq y)$ in $\mathbb{F}_{3^m}^*$.

(I) x, y are squares in $\mathbb{F}_{3^m}^*$. In this subcase, (10) becomes

$$\begin{cases} x + y + 1 = 0, \\ x^e + y^e + 1 = 0. \end{cases}$$

which leads to

$$1 + y^{3^h+2\cdot 3^i} = (y+1)^{3^h+2\cdot 3^i}. \tag{11}$$

Notice that

$$(y+1)^{3^h+2\cdot 3^i} = y^{3^h+2\cdot 3^i} - y^{3^h+3^i} + y^{3^h} + y^{2\cdot 3^i} - y^{3^i} + 1.$$

Then the Eq. (11) turns to

$$(y^{3^h} - y^{3^i})(y^{3^i} - 1) = 0.$$

It leads to

$$(y^{3^{h-i}} - y)^{3^i}(y^{3^i} - 1) = 0.$$

Then, we can get $y^{3^{h-i}} = y$ or $y^{3^i} = 1$, which implies that $y \in \mathbb{F}_3 = \{0, 1, -1\}$ since $\gcd(h-i, m) = 1$ and $(3^i, 3^m - 1) = 1$. Thus $y = -1$ since $y \neq 0$ and $y \neq 1$, we have $x = 0$. This is a contradiction to the assumption that $x \neq 0$. Therefore, $\omega \neq 3$.

(II) x is a square in $\mathbb{F}_{3^m}^*$ and y is a nonsquare in $\mathbb{F}_{3^m}^*$. Then (10) becomes

$$\begin{cases} x - y + 1 = 0, \\ x^e + y^e + 1 = 0. \end{cases}$$

which leads to

$$(y-1)^{3^h+2\cdot 3^i} + y^{3^h+2\cdot 3^i} + 1 = 0 \tag{12}$$

Notice that

$$(y-1)^{3^h+2\cdot 3^i} = y^{3^h+2\cdot 3^i} + y^{3^h+3^i} + y^{3^h} - y^{2\cdot 3^i} - y^{3^i} - 1.$$

This together with (12) yields

$$y^{3^h+2\cdot 3^i} = (y^{3^h} - y^{3^i})(y^{3^i} + 1).$$

Set $a = y^{-1} + 1$. Then we have

$$a^{3^i}(a^{3^i} - a^{3^h}) = 1. \tag{13}$$

By the Eqs. (8), (13) cannot hold for any $a \in \mathbb{F}_{3^m}^*$. Therefore, $\omega \neq 3$.

(III) x is a nonsquare in $\mathbb{F}_{3^m}^*$ and y is a square in$\mathbb{F}_{3^m}^*$. This case is similar to subcase (II). Therefore, $\omega \neq 3$.

(IV) x and y are nonsquares in$\mathbb{F}_{3^m}^*$. Then (10) becomes

$$\begin{cases} -x - y + 1 = 0, \\ x^e + y^e + 1 = 0. \end{cases}$$

which leads to

$$(x-1)^{3^h+2\cdot 3^i} = x^{3^h+2\cdot 3^i} + 1 \tag{14}$$

Notice that

$$(x-1)^{3^h+2\cdot 3^i} = x^{3^h+2\cdot 3^i} + x^{3^h+3^i} + x^{3^h} - x^{2\cdot 3^i} - x^{3^i} - 1.$$

This together with (14) yields

$$(x^{3^h} - x^{3^i})(x^{3^i} - 1) = -1.$$

Set $b = x + 1$. Then we have

$$b^{3^i}(b^{3^i} - b^{3^h}) = 1. \tag{15}$$

By the Eq. (8), the Eq. (15) cannot hold for any $b \in \mathbb{F}_{3^m}^*$. Therefore, $\omega \neq 3$.

Case B: $c_1 = c_2 = 1$ and $c_3 = -1$. In this case, we have

$$\begin{cases} x^t + y^t - 1 = 0, \\ x^e + y^e - 1 = 0. \end{cases} \tag{16}$$

Note that t is even and e is odd. Let $\bar{x} = -x$ and $\bar{y} = -y$. Then (16) becomes

$$\begin{cases} \bar{x}^t + \bar{y}^t - 1 = 0, \\ \bar{x}^e + \bar{y}^e + 1 = 0. \end{cases}$$

The rest of the proof of this case is similar to Case A. We omit the details here. Hence $\omega \neq 3$.

The discussion above shows that the $C_{(t,e)}$ does not have a codeword of Hamming weight $\omega \in \{1, 2, 3\}$. Hence $d \geq 4$. On the other hand, according to the sphere packing bound (see [2]), the minimum distance of any linear code of length $3^m - 1$ and dimension $3^m - 2m - 1$ should be less than or equal to 4. Hence $d = 4$. The proof is complete. □

Now we will present some new cyclic codes $C_{(t,e)}$ in the sequel by choosing different values of t and e. Let h be a positive integer and $m = 2h - 1$. Then

$$\gcd(h - i, m) = 1, i = 0, 1.$$

If $m \not\equiv 0 \pmod 3$, we have

$$\gcd(h - 2, m) = 1.$$

Using Lemma 4, Lemma 5 and Theorem 1, we have the following results.

Theorem 2. *Let $h > 1$ be a positive integer, $m = 2h - 1$ and t, e satisfying that (7). If $m \not\equiv 0 \pmod 3$ and*

1) $i = 0$, or
2) $i = 1$ and $h \not\equiv 3 \pmod 5$, or
3) $i = 2$ and $h \not\equiv 27 \pmod{53}$,

then the ternary cyclic code $C_{(t,e)}$ has parameters $[3^m - 1, 3^m - 2m - 1, 4]$.

It should be noted that $e = 3^h + 2 \cdot 3^i$ if $i = 0$, which is equivalent to $e = 3^h + 2$.This is a special case of Ref. [15].

Let k be a positive integer. The item 2) of Theorem 2 holds if one of the following cases holds:

(i) let $h = 3k + 1$ and $3k \not\equiv 2 \pmod 5$;

(ii) let $h = 3k + 3$ and $k \not\equiv 0 \pmod 5$.

The item 3) of Theorem 2 holds if one of the following cases holds:

(i) let $h = 3k + 1 > 1$ and $3k \not\equiv 26 \pmod{53}$;

(ii) let $h = 3k + 3$ and $k \not\equiv 8 \pmod{53}$.

The following examples from the Magma Program confirm Theorem 2.

Example 1. *Let $k = 0$, $h = 3k+3 = 3$, $m = 2h-1 = 5$ and ω be a generator of $\mathbb{F}_{3^m}^*$ with minimal polynomial $x^5 + 2x + 1$. Then $t = \frac{3^m+1}{2} = 122, e = 3^h + 18 = 45$. We have the code $C_{(t,e)}$ is an optimal ternary cyclic code with generator polynomial $x^{10} + 2x^9 + x^8 + x^7 + x^5 + x^3 + x^2 + x + 2$ and parameters* $[242, 232, 4]$.

Example 2. *Let $k = 1$, $h = 3k+1 = 4$, $m = 2h-1 = 7$ and ω be a generator of $\mathbb{F}_{3^m}^*$ with minimal polynomial $x^7 + 2x + 1$. Then $t = \frac{3^m+1}{2} = 1094, e = 3^h + 6 = 87$. We have the code $C_{(t,e)}$ is an optimal ternary cyclic code with generator polynomial $x^{14} + 2x^{13} + 2x^{12} + 2x^{11} + 2x^{10} + 2x^9 + 2x^8 + 2x^7 + 2x^4 + x^3 + 2$ and parameters* $[2186, 2172, 4]$.

4. Conclusion

In the letter, inspired by the work of [15], two new classes of optimal ternary cyclic codes were presented. It should be noticed that more new optimal ternary codes may be obtained from other values of $e = 3^h + 2 \cdot 3^i$.

References

[1] R. Lidl and H. Niederreiter, Finite Fields, Cambridge University Press, 1996.
[2] J. Lint, Introduction to Coding Theory, 3rd ed., Springer-Verlag Press, 1999.
[3] T. Kløve, Codes for Error Detection, World Scientific Press, 2007.
[4] C. Carlet, C. Ding, and J. Yuan, "Linear codes from perfect nonlinear mappings and their secret sharing schemes," IEEE Trans. Inf. Theory, vol.51, no.6, pp.2089–2102, 2005.
[5] C. Ding and X. Wang, "A coding theory construction of new systematic authentication codes," Theory Comput. Sci., vol.330, no.1, pp.81–99, 2005.
[6] C. Ding, Y. Yang, and X. Tang, "Optimal sets of frequency hopping sequences from linear cyclic codes," IEEE Trans. Inf. Theory, vol.56 no.7, pp.3605–3612, 2010.
[7] C. Ding and T. Helleseth, "Optimal ternary cyclic codes from monomials," IEEE Trans. Inf. Theory, vol.59, no.9, pp.5898–5904, 2013.
[8] N. Li, C. Li, T. Helleseth, C. Ding, and X. Tang, "Optimal ternary cyclic codes with minimum distance four and five," Finite Fields and Their Appl., vol.30, no.6, pp.100–120, 2013.
[9] N. Li, Z. Zhou, and T. Helleseth, "On a conjecture about a class of optimal ternary cyclic codes," 2015 Seventh International Workshop on Signal Design and its Applications in Communications (IWSDA), Bengaluru, India, pp.62–65, 2015.
[10] D. Han and H. Yan, "On an open problem about a class of optimal ternary cyclic codes," Finite Fields and Their Appl., vol.59, pp.335–343, 2019.
[11] Z. Zha and L. Hu, "New classes of optimal ternary cyclic codes with minimum distance four," Finite Fields and Their Appl., vol.64, 101671, 2020.
[12] L. Wang and G. Wu, "Several classes of optimal ternary cyclic codes with minimal distance four," Finite Fields and Their Appl., vol.40, pp.126–137, 2016.
[13] C. Fan, N. Li, and Z. Zhou, "A class of optimal ternary cyclic codes and their duals," Finite Fields and Their Appl., vol.37, pp.193–202, 2016.
[14] Z. Zhou and C. Ding, "A class of three-weight cyclic codes," Finite Fields and Their Appl., vol.25, pp.79–93, 2014.
[15] H. Yan, Z. Zhou, and X. Du, "A family of optimal ternary cyclic codes from the Niho-type exponent," Finite Fields and Their Appl., vol.54, pp.101–112, 2018.
[16] Z. Zha, L. Hu, and Y. Liu, "Further results on optimal ternary cyclic codes," Finite Fields and Their Appl., vol.75, 101898, 2021.
[17] Y. Liu, X. Cao, and W. Lu, "Two classes of new optimal ternary cyclic codes," Adv. Math. Commun., vol.17, no.4, pp.979–993, 2023.
[18] Z. Hai, L. Rong, and T. Sun, "Two families of optimal ternary cyclic codes with minimal distance four," Finite Fields and Their Appl., vol.79, 101995, 2022.
[19] D. Wang and X. Cao, "A family of optimal ternary cyclic codes with minimum distance five and their duals," Cryptogr. Commun., vol.14, no.1, pp.1–13, 2022.
[20] Q. Liu and X. Liu, "On some conjectures about optimal ternary cyclic codes," Appl. Algebra Eng. Commun. Comput., vol.33, no.4, pp.419–436, 2022.
[21] Q. Li and L. Liu, "Some results on optimal ternary cyclic codes with minimal distance four," Chinese Journal of Electronics, vol.33, pp.1–10, 2023.
[22] N. Li and S. Mesnager, "Recent results and problems on constructions of linear codes from cryptographic functions," Cryptogr. Commun., vol.12, no.5, pp.965–986, 2020.