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LETTER

A High-Performance Antenna Array Signal Processing Method in Deep Space Communication

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SUMMARY In future deep space communication missions, VLBI (Very Long Baseline Interferometry) based on antenna array technology remains a critical detection method, which urgently requires the improvement of synthesis performance for antenna array signals. Considering this, focusing on optimizing the traditional antenna grouping method applied in the phase estimation algorithm, this letter proposes a "$L/2$ to $L/2$" antenna grouping method based on the maximum correlation signal-to-noise ratio (SNR). Following this idea, a phase difference estimation algorithm named "Couple" is presented. Theoretical analysis and simulation verification illustrate that: when $\rho < -10\,dB$, the proposed "Couple" has the highest performance; increasing the number of antennas can significantly improve its synthetic loss performance and robustness. The research of this letter indicates a promising potential in supporting the rising deep space exploration and communication missions.
key words: *deep space exploration and communication, maximum correlation signal-to-noise ratio (SNR), subband phase difference estimation*

1. Introduction

With the growing demand for high-rate and long-distance space communication, the signal bandwidth of deep space spacecraft gets larger [1]. The deep space communication system based on antenna array technology and Very Long Baseline Interferometry (VLBI) [2] thus urgently needs a method that can effectively receive wideband low signal-to-noise ratio (SNR) signals. The subband phase difference estimation technique is a core technology of the frequency domain synthesis method, whose performance directly affects the performance of residual delay and phase estimation [3]. Therefore, in future deep space communication-related missions, to improve the performance of antenna array signal synthesis under low SNR, the superior subband phase difference estimation technique is of utmost importance [4].

The estimation performance of phase difference mainly depends on the correlation SNR, which refers to the signal SNR of two signals after correlation processing. This includes conjugate multiplication and cumulative average, which mainly depends on the integration time, correlation bandwidth, and the SNR of correlated signals. As the former two elements are limited by objective conditions such as atmospheric coherence time, improving the SNR of the two related signals is meaningful in effectively enhancing the synthesis performance of antenna array signals [5].

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The SNR of the correlation signal in antenna arrays depends on the antenna grouping method applied in the phase estimation algorithm [6]. Currently, the most widely used algorithms include Simple algorithm, Sumple algorithm, and Matrix-free algorithm, with different antenna grouping methods: Simple algorithm adopts the "1 to 1" method, which has the worst performance; Sumple algorithm adopts the "1 to $L-1$" method (L is the number of antennas), which remains the most comprehensive performance among existing methods; Matrix-free algorithm adopts "1 to L" method, with slow convergence rate and worse performance than Sumple algorithm due to its existence of autocorrelation component [7].

Based on the above analysis, in this letter, a "$L/2$ to $L/2$" antenna grouping method and a phase difference estimation algorithm called "Couple" are proposed based on the idea of maximum correlation SNR, which will be shown in Sect. 2. To the best of our knowledge, this is the first time such a method to apply in phase difference estimation of antenna array signals, with a performance better than any existing algorithm, especially in convergence characteristics and synthesis loss. In Sect. 3, the performance of traditional and proposed algorithms are compared through experimental simulation, which further proves the superiority of the proposed method. Section 4 provides the conclusive remark and outlook of this letter.

Denotion $g_i(t_k)$: related signals; ϕ_i: mean values of phases of each element in $g_i(t_k)$; Z_K: signals after accumulated and averaged; $\overline{Z_S}$: valuable signal component conditional on ϕ_{ij}; $\overline{Z_K}$: signals after the cumulative average of the correlation; $\overline{Z_N}$: noise component conditional on ϕ_i and ϕ_j; γ: SNR of $\overline{Z_K}$; σ^2_ϕ: estimated variance of ϕ; L,M: number of antennas; $\Gamma(\gamma)$: phase estimation loss factor.

2. System Model

2.1 Construction of Phase Difference Estimation Model Based on "Many-to-Many" Antenna Grouping Method

Figure 1 shows the phase difference estimation model based on the "many-to-many" antenna grouping method. The phase difference estimated by this model is called the relevant phase as it is not the final phase correction value. Considering the uniform array, we assume that there are a total of L antennas (each antenna signal is a complex signal), the bandwidth is B, and the related signals are $g_1(t_k)$ and $g_2(t_k)$ respectively. Then we have:

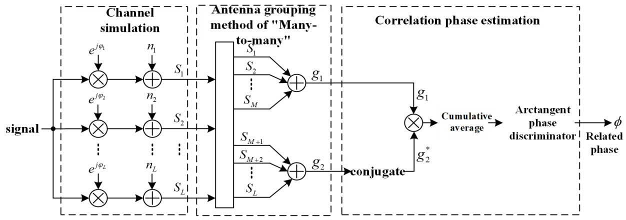

Fig. 1 Phase difference estimation model based on "many-to-many" antenna grouping method.

$$\begin{cases} g_1(t_k) = \sum\limits_{i=1}^{M} \left(\sqrt{P} e^{j(\omega t_k + \varphi_i)} + n_i(t_k)\right) \\ g_2(t_k) = \sum\limits_{j=M+1}^{L} \left(\sqrt{P} e^{j(\omega t_k + \varphi_j)} + n_j(t_k)\right) \end{cases} \tag{1}$$

Where constant P is the signal power of each antenna, ω is the intermediate corner frequency of the signal, φ_i and $n_i(t_k)$ are the initial phase of the signal and complex Gaussian random noise of the antenna i. $n_i(t_k)$ obeys $N(0, \sigma^2)$. The signal and noise, and the noise of different antennas are assumed to be independent of each other. To ensure that $g_1(t_k)$ and $g_2(t_k)$ have no autocorrelation components, the number of antennas contained by them is M and $L - M$ without common terms.

To reflect the phase difference between $g_1(t_k)$ and $g_2(t_k)$, let the mean values of phases of each element in $g_1(t_k)$ and $g_2(t_k)$ be ϕ_1 and ϕ_2, respectively, and then we have:

$$\phi_1 = \frac{1}{M} \sum_{i=1}^{M} \varphi_i \tag{2}$$

$$\phi_2 = \frac{1}{L-M} \sum_{j=M+1}^{L} \varphi_j \tag{3}$$

Thus, the distribution characteristics of phases φ_i and φ_j of each element in $g_1(t_k)$ and $g_2(t_k)$ are given. Noticing that M and L determine the antenna grouping method: when $M = 1$ and $L = 2$, Eq. (1) can represent Simple algorithm; while $M = 1$ and $L > 2$, Eq. (1) represents Sumple algorithm; and when $M = L/2$ and $L > 2$, Eq. (1) will represent the proposed Couple algorithm, whose excellent performance will be demonstrated later.

2.2 Theoretical Analysis of the Relative Phase Estimation Performance of Couple Algorithm

As shown in Fig. 1, a cross-correlation operation is performed on $g_1(t_k)$ and $g_2(t_k)$. Since the independent assumption of signal and noise, and the noise of different antennas, the SNR of the related signal $Z(t_k)$ can be improved by accumulative averaging. After accumulative averaging, noise power as a random quantity can be transformed into a deterministic quantity. Considering this, in the letter, we define the cumulative averaged equivalent noise values for this period($K = BT$) are: n_{iK}(for $n_i(t_k)$) and n_{jK}(for $n_j(t_k)$), respectively. Thus we have: $\frac{1}{K}\sum\limits_{l=1}^{K} n_j^*(t_k) = \frac{1}{\sqrt{K}} n_{jK}^*$, $\frac{1}{K}\sum\limits_{l=1}^{K} n_i(t_k) = \frac{1}{\sqrt{K}} n_{iK}$, $\frac{1}{K}\sum\limits_{l=1}^{K} n_i(t_k) n_j^*(t_k) = \frac{1}{\sqrt{K}} n_{iK} n_{jK}^*$. Let the signal after accumulated and averaged be Z_K, then we can obtain:

$$\begin{aligned} Z_K &= \frac{1}{K}\sum_{l=1}^{K} (Z(t_k)) \\ &= P\sum_{i=1}^{M}\sum_{j=M+1}^{L}\left[e^{j(\varphi_i-\varphi_j)}\right] + \sqrt{\frac{P}{K}}\sum_{i=1}^{M}\sum_{j=M+1}^{L}\left[e^{j(\omega t_k+\varphi_i)} n_{jK}^*\right] + \\ &\sqrt{\frac{P}{K}}\sum_{i=1}^{M}\sum_{j=M+1}^{L}\left[e^{-j(\omega t_k+\varphi_j)} n_{iK}\right] + \sqrt{\frac{1}{K}}\sum_{i=1}^{M}\sum_{j=M+1}^{L}\left[n_{iK} n_{jK}^*\right] \end{aligned} \tag{4}$$

Let $\phi_{ij} = \varphi_i - \varphi_j$, then $\phi_{ij} \sim N(\phi_1 - \phi_2, 2\sigma_\phi^2)$ as φ_i and φ_j are independent. Let the signal in Z_K as Z_S, then we can obtain: $Z_S = P\sum\limits_{i=1}^{M}\sum\limits_{j=M+1}^{L}\left[e^{j\phi_{ij}}\right]$.

Let the valuable signal component conditional on ϕ_{ij} be $\overline{Z_S}$. According to the characteristic function formula of Gaussian random variable[8], we have: $E\left[e^{j\phi_{ij}}\left|\phi_{ij}\right.\right] = e^{j\phi - \frac{1}{2}\left(2\sigma_\phi^2\right)} = e^{j\phi} e^{-\sigma_\phi^2}$. Where σ_ϕ^2 reflects the phase alignment degree of each element signal. After convergence in the iterative process, $e^{-\sigma_\phi^2} \approx 1$, thus we have: $E\left[e^{j\phi_{ij}}\left|\phi_{ij}\right.\right] = e^{j\phi}$. Where: ϕ is the estimated related phase, and its physical meaning is:

$$\phi = \phi_1 - \phi_2 = \frac{1}{M}\sum_{i=1}^{M} \varphi_i - \frac{1}{L-M}\sum_{j=M+1}^{L} \varphi_j \tag{5}$$

Thus $\overline{Z_S}$ can be simplified as:

$$\overline{Z_S} = M(L-M) P e^{j\phi} \tag{6}$$

According to Eqs. (4), (5) and (6), the signal after the cumulative average of the correlation is:

$$\begin{aligned} \overline{Z_K} = M(L-M)Pe^{j\phi} &+ \sqrt{\frac{P}{K}} M e^{j(\omega t_k+\phi_1)} \sum_{j=M+1}^{L} n_{jK}^* \\ &+ \sqrt{\frac{P}{K}} (L-M) e^{-j(\omega t_k+\phi_2)} \sum_{i=1}^{M} n_{iK} \\ &+ \sqrt{\frac{1}{K}} \sum_{i=1}^{M}\sum_{j=M+1}^{L}\left[n_{iK} n_{jK}^*\right] \end{aligned} \tag{7}$$

Next, the correlation SNR will be derived according to $\overline{Z_K}$, then the variance performance of the correlation phase ϕ will be analyzed. Firstly, the signal and noise power of $\overline{Z_K}$ will be analyzed respectively. The signal power in $\overline{Z_K}$ is:

$$\begin{aligned} S &= E\left(\overline{Z_K}\right) E\left(\overline{Z_K^*}\right) \\ &= M^2 (L-M)^2 P^2 \end{aligned} \tag{8}$$

Since the noise of each antenna is a complex random variable, and the variance of real and imaginary parts is σ^2, then we can get:var $\left(n_{iK}^*\right) = \text{var}\left(n_{jK}^*\right) = 2\sigma^2$. Thus the noise power in $\overline{Z_K}$ is:

$$N = \frac{2\sigma^2}{K}\left(LP + 2\sigma^2\right)\left[M\left(L-M\right)\right] \tag{9}$$

According to Eqs. (8) and (9), the SNR of $\overline{Z_K}$, γ can be expressed as:

$$\begin{aligned}\gamma &= \frac{S}{N} = \frac{M^2(L-M)^2P^2}{\frac{2\sigma^2}{K}\left(LP+2\sigma^2\right)\left[M(L-M)\right]} \\ &= \frac{KP^2}{2\sigma^2\left(LP+2\sigma^2\right)}\left[M\left(L-M\right)\right]\end{aligned} \tag{10}$$

Set the SNR of single antenna as ρ, then $\rho = P/(2\sigma^2)$. Substituting it into Eq. (10) we get:

$$\gamma = \frac{K\rho^2}{1+L\rho}\left[M\left(L-M\right)\right] \tag{11}$$

It can be seen from Eq. (11) that under the condition of constant accumulation times K and single antenna SNR ρ. As different antenna group methods (M is different) have different relevant SNR, Eq. (11) can be derived as follows:

$$\gamma \leq \frac{K\rho^2}{1+L\rho}\left[\frac{M^2+(L-M)^2}{2}\right] \tag{12}$$

According to the property of Eq. (12), the more equal the number of array elements is divided, the greater the correlation SNR. Thus we have:

$$M = \begin{cases}\frac{L_{even}}{2} \\ \frac{L_{odd}\pm 1}{2}\end{cases} \tag{13}$$

When array antennas are allocated according to Eqs. (13) and (14), the maximum correlation SNR can be achieved respectively:

$$\gamma_{\max} = \begin{cases}\frac{K{L_{even}}^2\rho^2}{4(1+L_{even}\rho)} \\ \frac{K\left({L_{odd}}^2-1\right)\rho^2}{4(1+L_{odd}\rho)}\end{cases} \tag{14}$$

When L is large, the influence of its parity on $\gamma_{\max}$ is basically negligible. Therefore, for the convenience of research, the default antenna number in this letter is even, and the antenna grouping method of "$L/2$ to $L/2$" is adopted, which we call it as "Couple" algorithm.

The above mathematical derivation proves that when the antenna grouping method of "$L/2$ to $L/2$" is adopted, the maximum correlation SNR can be achieved, and the estimated variance of the correlation phase ϕ can be obtained as:

$$\sigma_\phi^2 = \frac{\Gamma(\gamma)}{2\gamma} = \frac{2\left(1+L\rho\right)}{KL^2\rho^2}\Gamma\left(\frac{KL^2\rho^2}{4\left(1+L\rho\right)}\right) \tag{15}$$

Where $\Gamma(\gamma)$ is the phase estimation loss factor, which is defined as follows:

Table 1 Comparison of relative phase estimation performance of Simple, Sumple and Couple algorithms.

Algorithm	Correlation SNR	Variance
Simple	$\frac{K\rho^2}{1+2\rho}$	$\frac{1+2\rho}{2K\rho^2}\Gamma\left(\frac{K\rho^2}{1+2\rho}\right)$
Sumple	$\frac{K(L-1)\rho^2}{1+L\rho}$	$\frac{1+L\rho}{2K(L-1)\rho^2}\Gamma\left(\frac{K(L-1)\rho^2}{1+L\rho}\right)$
Couple	$\frac{KL^2\rho^2}{4(1+L\rho)}$	$\frac{2(1+L\rho)}{KL^2\rho^2}\Gamma\left(\frac{KL^2\rho^2}{4(1+L\rho)}\right)$

$$\Gamma(\gamma) = 2\gamma\int_{-\pi}^{\pi}\Delta\theta_e^2 f\left(\Delta\theta_e,\gamma\right)d\Delta\theta_e \tag{16}$$

Where $\Delta\theta$ is the phase difference to be estimated, $\Delta\theta_e$ is the estimation error of $\Delta\theta$, $f\left(\Delta\theta_e,\gamma\right)$ is the probability density function of $\Delta\theta_e$.

2.3 Comparison of the Relative Phase Estimation Performance of Proposed Couple Algorithm and Traditional Algorithms

Next, to prove the superiority of the proposed Couple algorithm, it is compared with traditional antenna grouping methods (Simple and Sumple). The estimation performance of the related phases of the proposed Couple and traditional algorithms is summarized in Table 1.

3. Simulation Results

3.1 Simulation Parameter Setting

In this experiment, the overall parameter setting is shown in Table 2. Based on this settting, the effectiveness and superiority of the proposed antenna grouping method and Couple algorithm are verified and analyzed.

3.2 Comparison of the Relative Phase Estimation Performance of Proposed Couple and Traditional Algorithms

As shown in Fig. 2(a) and (b), the Couple algorithm is used for phase estimation, the estimated variance and estimated loss factor of the related phases are calculated, and the simulation results are compared with the theoretical values. Theoretical values of the variance of correlation phase estimation (which is illustrated in Fig. 2(a)) are obtained from Eq. (15); while those of the phase estimation loss factor are obtained from Eq. (16). The CRLB of $\Delta\theta$ is expressed as follows:

$$\text{CRLB}\left(\Delta\theta\right) = \frac{1}{2\gamma} \tag{17}$$

It can be seen that the simulation results obtained by the Couple algorithm are in good agreement with the theoretical values, especially when the estimated variance is less than 1, which verifies the correctness of the theoretical derivation.

Figure 3(a) and (b) show the relevant phase estimation variance performance and the corresponding performance of the phase estimation loss factor of the three algorithms under different SNRs and antenna numbers. We can find

Table 2 Overall parameter setting of the simulation.

Signal type	sampling number of each symbol
BPSK	10
initial phase	average number of accumulation
0	1000
SNR of a single antenna	total number of antennas
$[-35dB, -5dB]$	36, 100
Monte Carlo simulation times	antenna array shape
10000	a uniform array

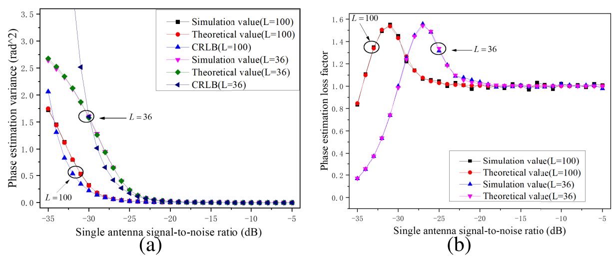

Fig. 2 Theoretical and simulation results of the correlation phase estimation performance of Couple algorithm. (a) Variance of correlation phase estimation. (b) Phase estimation loss factor.

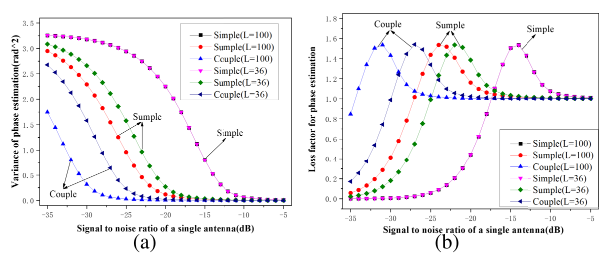

Fig. 3 Comparison of phase estimation performance of the traditional and proposed algorithms. (a) Variance performance of phase estimation. (b) Performance of phase estimation loss factor.

that: (i) When $\rho < -10\,dB$, the proposed Couple algorithm has the highest performance, followed by Sumple and Simple algorithm. (ii) The performance of both Sumple and Couple algorithms can be improved by increasing the number of antennas (Couple performs better estimation than Sumple), while Simple algorithm cannot.

Finally, as shown in Fig. 4, we investigate the synthetic loss convergence performance of the algorithms under different antenna numbers and single antenna SNRs. The value of SNR in the evaluation for Fig. 4(a) is $-21\,dB$, and it shows that increasing the number of antennas can significantly improve the synthetic loss performance and robustness of the algorithm, with Couple algorithm performs significantly better, but it has little effect on the convergence rate. Figure 4(b) shows that with the increase in the number of iterations, the synthetic loss of Couple algorithm will remain within $-1.5\,dB$.

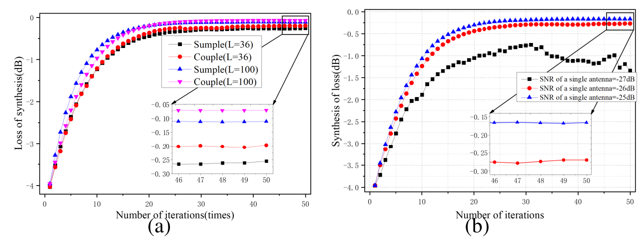

Fig. 4 Synthesis loss convergence performance of the algorithms with different antenna numbers and single antenna SNRs. (a) Performance of the algorithms with different antenna numbers. (b) Performance of the algorithms with different single antenna SNRs.

4. Conclusion

Facing the urgent demand for high-performance antenna array signal processing in future deep space communication, this letter proposes a phase difference estimation algorithm with the characteristic of maximum correlation SNR. Through theoretical derivation and simulations, the phase difference estimation performance, convergence characteristics, and synthesis performance of traditional algorithms and the proposed algorithm are compared, verifying the effectiveness and superiority of the proposed method, which could support the increasingly frequent deep space exploration missions in the future.

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