LETTER

# Pre-T Event-Triggered Controller with a Gain-Scaling Factor for a Chain of Integrators and Its Extension to Strict-Feedback Nonlinearity*

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**SUMMARY** We propose a pre-*T* event-triggered controller (ETC) for the stabilization of a chain of integrators. Our per-T event-triggered controller is a modified event-triggered controller by adding a pre-defined positive constant *T* to the event-triggering condition. With this pre-T, the immediate advantages are (i) the often complicated additional analysis regarding the Zeno behavior is no longer needed, (ii) the positive lower bound of interexecution times can be specified, (iii) the number of control input updates can be further reduced. We carry out the rigorous system analysis and simulations to illustrate the advantages of our proposed method over the traditional event-triggered control method.

***key words:*** *pre-T event-triggered controller, a chain of integrators, global stabilization*

## 1. Introduction

Since the work of [12], the event-triggered control (ETC) has been one of much studied topics in control field (see [3], [5], [8]–[13] and references therein). The most well-known advantage of the traditional ETCs is the discrete updates of control inputs, which leads to the efficient usage of communication resources. The often difficult part of the traditional ETCs is the design of the event-triggering conditions because the event-triggering conditions must be designed to be fit in both the system stabillization/regulation analysis and interexecution time analysis to avoid the Zeno behavior [6].

In this letter, we consider a chain of integrators and take the ETC with a gain-scaling factor from [11] and modify it by adding a pre-defined positive constant *T* to the event-triggering conditions. So, we call it 'a pre-T ETC with a gain-scaling factor'. This pre-T approach is motivated by a zero-order-hold control method in [1]. So, our pre-T ETC can be considered as a hybrid version of zero-order-hold and ETC control methods.

There are certain advantages with our proposed pre-T ETC. First, the positive lower bounds of intereexecution times are guaranteed by default. Thus, there is no need to carry out the analysis to show the avoidance of the Zeno behavior. Second, since the event-triggering conditions are only used in the system stabilization/regulation analysis, the design of the event-triggering conditions can be easier - we do not have to consider two-way analyses unlike the traditional ETCs. Third, the traditional ETCS tend to yield smaller interexecution times during the transient period. In our proposed control method, there are fixed lower bound *T* such that somewhat unnecessarily small interexecution bounds can be avoided during the transient period, which leads to even further reduced number of control input updates over the traditional ETCs. We carry out the system analysis and show simulation results which clearly show the validity of our proposed control method.

## 2. System and problem formulation

We consider a chain of integrators given by

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}$ is the input, and $(A, B)$ is a Brunovsky canonical pair, that is, $A = [a_{ij}]$, $1 \le i, j \le n$, where if $j = i + 1$, $a_{ij} = 1$, else $a_{ij} = 0$ and $B = [0, \cdots, 0, 1]^T$.

Our control goal is to globally asymptotically stabilize the system (1) within a framework of ETC. More specifically, we suggest a new ETC with an additional pre-T feature to further reduce the number of control input updates while maintaining the control performance. Our proposed pre-T ETC is given as

$$u = K_\gamma x(t_i), \ \ \forall t \in [t_i, t_{i+1}) \tag{2}$$

and a triggering condition with a pre-T is

$$\begin{aligned} t^*_{i+1} &= \inf\{t > t_i : \|E_\gamma e\| > \sigma \|E_\gamma x\|\} \\ t^{**}_{i+1} &= T + t_i \\ t_{i+1} &= \max\{t^*_{i+1}, t^{**}_{i+1}\} \end{aligned} \tag{3}$$

where $K_\gamma = [k_1/\gamma^n, \cdots, k_n/\gamma]$, $E_\gamma = \text{diag}[\gamma, \cdots, \gamma^n]$, $e = x(t_i) - x$, $t_0 = 0$.

The following controller parameters with brief explanations are to be chosen.

- $K = K_\gamma|_{\gamma=1}$ Basic control gain
- $\gamma \ge 1$ Gain-scaling factor
- $0 < \sigma < 1$ Triggering condition factor

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• $0 < T$ Pre-defined lower bound of interexcecution times

Without any further notice, $\|\cdot\|$ denotes the Euclidean norm throughout this letter.

## 3. Main Results

**Theorem 1.** *The system* (1) *is globally asymptotically stabilized by the controller* (2) *with* (3).

*Proof.* For $t \in [t_i, t_{i+1})$, the closed-loop system is

$$\dot{x} = A_{K_\gamma} x - BK_\gamma (x - x(t_i)) \tag{4}$$

where $A_{K_\gamma} = A + BK_\gamma$.

First, we let $K$ be that $A_K = A_{K_\gamma}|_{\gamma=1}$ is Hurwitz. Then, we have the following Lyapunov equation [11]

$$A_{K_\gamma}^T P_\gamma + P_\gamma A_{K_\gamma} = -\gamma^{-1} E_\gamma^2 \tag{5}$$

where $P_\gamma = E_\gamma P E_\gamma$, $A_K^T P + PA_K = -I$.

Here, we consider the following two cases.

***Case 1.*** $t_{i+1} = t_{i+1}^*$.

In this case, the closed-loop system is rewritten as

$$\dot{x} = A_{K_\gamma} x + BK_\gamma e \tag{6}$$

With $V(x) = x^T P_\gamma x$, along the trajectory of (6),

$$\begin{aligned}\dot{V}(x) &= -\gamma^{-1}\|E_\gamma x\|^2 + 2x^T P_\gamma BK_\gamma e \\ &= -\gamma^{-1}\|E_\gamma x\|^2 + 2x^T E_\gamma P E_\gamma BK_\gamma E_\gamma^{-1} E_\gamma e\end{aligned} \tag{7}$$

Using $E_\gamma BK_\gamma E_\gamma^{-1} = \gamma^{-1}BK$ and $\|E_\gamma e\| \le \sigma \|E_\gamma x\|$ for $t \in [t_i, t_{i+1})$, we have

$$\begin{aligned}\dot{V}(x) &\le -\gamma^{-1}\|E_\gamma x\|^2 + 2\|P\|\|BK\|\sigma\gamma^{-1}\|E_\gamma x\|^2 \\ &= -\gamma^{-1}(1 - 2\|P\|\|BK\|\sigma)\|E_\gamma x\|^2\end{aligned} \tag{8}$$

By choosing $\sigma$ as

$$\sigma \ge \frac{1-c}{2\|P\|\|BK\|} > 0, \;\; 0 < c < 1 \tag{9}$$

we have

$$\dot{V}(x) \le -\gamma^{-1} c \|E_\gamma x\|^2 \tag{10}$$

***Case 2.*** $t_{i+1} = t_{i+1}^{**}$. In this case, the closed-loop system is rewritten as

$$\begin{aligned}\dot{x} =& A_{K_\gamma} x - BK_\gamma \int_{t_i}^{t} \dot{x} ds \\ =& A_{K_\gamma} x - BK_\gamma \int_{t_i}^{t} [Ax + BK_\gamma x(t_i)] ds \\ =& A_{K_\gamma} x - BK_\gamma A \int_{t_i}^{t} x ds - (BK_\gamma)^2 \int_{t_i}^{t} x(t_i)] ds \\ =& A_{K_\gamma} x - BK_\gamma A E_\gamma^{-1} \int_{t_i}^{t} E_\gamma x ds \\ & - (BK_\gamma)^2 E_\gamma^{-1} \int_{t_i}^{t} E_\gamma x(t_i)] ds\end{aligned} \tag{11}$$

With $V(x) = x^T P_\gamma x$, along the trajectory of (11),

$$\begin{aligned}\dot{V}(x) =& -\gamma^{-1}\|E_\gamma x\|^2 \\ & - 2x^T E_\gamma P E_\gamma BK_\gamma A E_\gamma^{-1} \int_{t_i}^{t} E_\gamma x ds \\ & - 2x^T E_\gamma P E_\gamma (BK_\gamma)^2 E_\gamma^{-1} \int_{t_i}^{t_i} E_\gamma x(t_i)] ds\end{aligned} \tag{12}$$

Note that $\|E_\gamma BK_\gamma A E_\gamma^{-1}\| \le \gamma^{-2}\|K\|$, $\|E_\gamma (BK_\gamma)^2 E_\gamma^{-1}\| \le \gamma^{-2}\|\bar{K}\|$ with $\bar{K} = k_n K$. Then, from (12), we have

$$\begin{aligned}\dot{V}(x) \le& -\gamma^{-1}\|E_\gamma x\|^2 \\ & + 2\|P\|\|K\|T\gamma^{-2}\|E_\gamma x\| \sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\| \\ & + 2\|P\|\|\bar{K}\|T\gamma^{-2}\|E_\gamma x\| \sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\|\end{aligned} \tag{13}$$

Here, we utilize the Razumikhin theorem [4] to treat the the supremum term. Setting $V(x(t+\theta)) \le qV(x)$, $-T \le \theta \le 0$ leads to $\sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\| \le \bar{q}\|E_\gamma x\|$, $\bar{q} = \sqrt{q\lambda_{\max}(P)/\lambda_{\min}(P)}$. Using this, we have

$$\dot{V}(x) \le -\gamma^{-1}[1 - 2\bar{q}\|P\|(\|K\| + \|\bar{K}\|)T\gamma^{-1}]\|E_\gamma x\|^2 \tag{14}$$

By choosing $\gamma$ as

$$\gamma \ge \frac{2\bar{q}\|P\|(\|K\| + \|\bar{K}\|)T}{1-c} > 0, \;\; 0 < c < 1 \tag{15}$$

we have

$$\dot{V}(x) \le -\gamma^{-1} c \|E_\gamma x\|^2 \tag{16}$$

which is the same as (10).

From (10) and (16) together, the closed-loop system (4) is globally asymptotically stabilized. Note that $T$ guarantees the presence of the positive lower bounds of interexecution times by default. So, unlike the traditional ETC methods, we do not need to carry out any additional analysis to prove the avoidance of the Zeno behavior. □

**Remark 1.** *The advantages of our proposed pre-T ETC over the traditional ETCs (see [11], [12]) are: (i) The traditional ETCs tend to have smaller interexecution times during the transient period as states often change rapidly. With fixed value of pre-T, our method can yield more regular and larger interexecution times during the transient period over the traditional ETCs; (ii) As stated at the end of proof of Theorem 1, we do not need to carry out the Zeno behavior avoidance analysis. This naturally leads to easier design of the event-triggering conditions because the event-triggering conditions are only involved in the system analysis, not Zeno behavior avoidance analysis.*

## 4. Extension to a Class of Strict-Feedback Nonlinearity

We extend the system (1) by adding a class of strict-feedback nonlinearity which represents of several practical systems such as a single-link robot manipulator and so forth [7] as follows.

$$\dot{x} = Ax + Bu + \delta(t,x,u) \tag{17}$$

where $\delta(t,x,u) = [\delta_1(t,x,u), \cdots, \delta_n(t,x,u)]^T$ is a vector of continuous functions under the following condition.

**Assumption 1.** *For $i = 1, \cdots, n$, there exists a finite constant $L \geq 0$ such that $\delta_i(t,x,u) \leq L(|x_1| + \cdots + |x_i|)$.*

Under Assumption 1, the analysis of the system (17) with our proposed controller (2) under (3) can be similarly derived by following the proof of Theorem 1 as follows:

***Case 1.*** $t_{i+1} = t^*_{i+1}$.

In this case, by adding a term of $2x^T P_\gamma \delta(t,x,u) \leq 2\|P\|L\|E_\gamma x\|^2$ to (8), we can have

$$\dot{V}(x) \leq -\gamma^{-1}\Delta_1(\sigma,\gamma)\|E_\gamma x\|^2 \tag{18}$$

where

$$\Delta_1(\sigma,\gamma) = 1 - 2\|P\|\|BK\|\sigma - 2\|P\|L\gamma \tag{19}$$

***Case 2.*** $t_{i+1} = t^{**}_{i+1}$.

In this case, again by adding a term of $2x^T P_\gamma \delta(t,x,u) \leq 2\|P\|L\|E_\gamma x\|^2$ to (14), we can have

$$\dot{V}(x) \leq -\gamma^{-1}\Delta_2(T,\gamma)\|E_\gamma x\|^2 \tag{20}$$

where

$$\Delta_2(T,\gamma) = 1 - 2\bar{q}\|P\|(\|K\| + \|\bar{K}\|)T\gamma^{-1} - 2\|P\|L\gamma \tag{21}$$

By summarizing (18)–(21), we arrive at the following result.

**Corollary 1.** *The system* (17) *is globally asymptotically stabilized by the controller* (2) *with* (3) *when a triplet* $(\sigma, T, \gamma)$ *is selected as* $\Delta_1(\sigma,\gamma) \geq c > 0$ *and* $\Delta_2(\sigma,\gamma) \geq c > 0$.

**Remark 2.** *Since each $\Delta_1(\sigma,\gamma)$ and $\Delta_2(\sigma,\gamma)$ consists of two parameters, we may need to use two contour maps containing two variables in order to determine the values of $(\sigma, T, \gamma)$. This parameter selection will be illustrated in the case 2 of the examples.*

## 5. Illustrative Examples

*Case 1: chain of integrators:* We consider a second-order system which represents a rigid body motion of satellite [2]. We select $K = [-1, -2]$, $\sigma = 0.05$, $\gamma = 7$, and $T = 0.8$. We make a comparison of our control method with [11] by using the same $K, \sigma, \gamma$. As shown in Figs. 1 and 2, the state trajectories generated by both methods are very similar, yet our method yields slightly better $x_1$ trajectory. The notable difference is that our method yields much reduced number of control input updates which is 108 times compared to 176 times by [11]. Thus, the number of control input updates

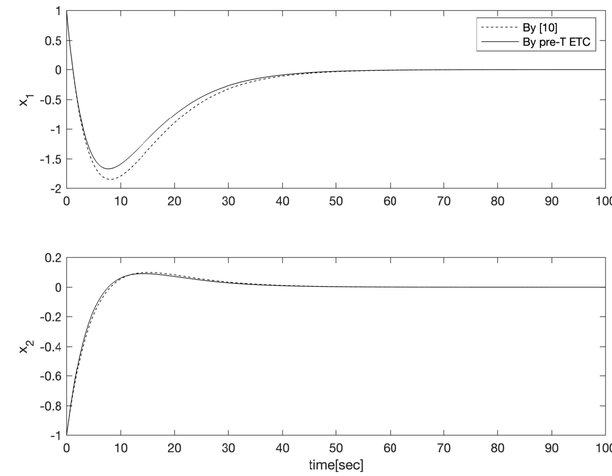


**Fig. 1** Comparison of state trajectories by [11] and pre-T ETC.

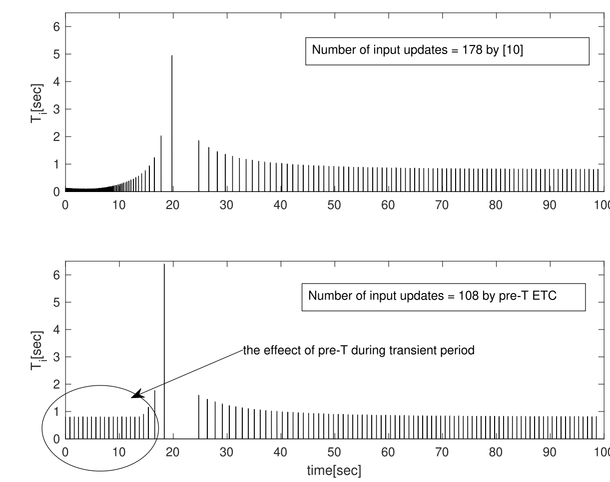


**Fig. 2** Comparison of number of input updates by [11] and pre-T ETC.

is reduced roughly by 39%. This result is made possible because our pre-T plays a role to prevent the interexecution times from becoming too small during the transient period as shown in Fig. 2. In summary, the pre-T mainly operates during the rapidly-varying transient period and once the states enter the steady-state mode, the traditional event-triggering condition dominates the controller operation. These combined approach makes our proposed controller generate less number of control input updates over the traditional ETCs while keeping the control performance almost the same.

*Case 2: Strict-feedback nonlinearity:* We consider a second-order system with $\delta(t,x,u) = [0, \omega(t)\sin x_1]$, $|\omega(t)| \leq 0.1$. We select $K = [-1, -2]$. With $K$, we can obtain two contour maps from (19) and (21) as follows:

From Fig. 3, we choose $(\sigma, T, \gamma) = (0.06, 0.02, 1.3)$ to complete the controller design. Its control results are shown in Fig. 4.

## 6. Conclusions

We have proposed a new pre-T ETC for a chain of integra-

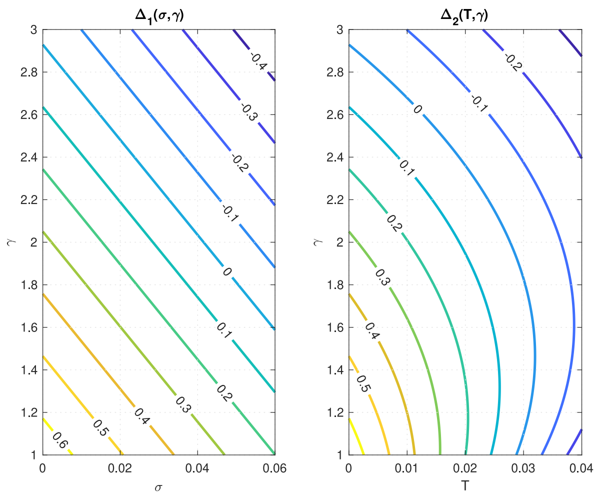


**Fig. 3** Contour maps of $\Delta_1(\sigma, \gamma)$ and $\Delta_2(T, \gamma)$.

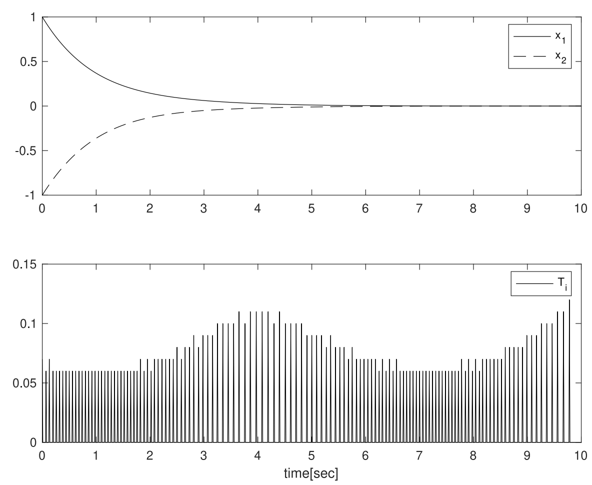


**Fig. 4** Control results by pre-T ETC.

tors. As shown in the analysis and simulation results, our method produces very similar control performance to the traditional ETCs while yielding much less number of control input updates. Since the main advantage of the ETCs are the discrete updates of control inputs, our pre-T ETC has the meaningful advantages over the traditional ETCs. Then, it has been shown that our control method can be extended to a class of strict-feedback nonlinear system. There can be much more extensions of current control method in future research.

## References

[1] H.-L. Choi, "Zero-order-hold triggered control of a chain of integrators with an arbitrary sampling period," IEICE Trans. Fundamentals, vol.E107-A, no.8, to appear, 2024.

[2] G. Franklin, J. Powell, and A. Emami-Naeini, Feedback Control of Dynamic Systems, 8th ed., Prentice Hall, Upper Saddle River, 2020.

[3] A. Girard, "Dynamic triggering mechanisms for event-triggered control," IEEE Trans. Autom. Control, vol.60, no.7, pp.1992–1997, 2015.

[4] J.K. Hale and S.M. Verduyn Lunel, Introduction to Funcitonal Differential Equations, Springer-Verlag, New York, 1993.

[5] J. Huang, W. Wang, C. Wen, and G. Li, "Adaptive event-triggered control of nonlinear systems with controller and parameter estimator triggering," IEEE Trans. Autom. Control, vol.65, no.1, pp.318–324, 2020.

[6] K. Johansson, M. Egerstedt, J. Lygeros, and S. Sastry, "On the regularization of zeno hybrid automata," Systems and Control Lett., vol.38, no.3, pp.141–150, 1999.

[7] H.K. Khalil, Nonlinear Systems, 3rd ed., Prentice Hall, Upper Saddle River, NJ 07458, 2002.

[8] F. Li and Y. Liu, "Adaptive event-triggered output-feedback controller for uncertain nonlinear systems," Automatica, vol.117, p.109006, 2020.

[9] D. Liang and J. Huang, "Robust output regulation of linear systems by event-triggered dynamic output feedback control," IEEE Trans. Autom. Control, vol.66, no.5, pp.2415–2422, 2021.

[10] F. Mazenc, M. Malisoff, and C. Barbalata, "Event-triggered control for continuous-time linear systems with a delay in the input," Systems and Control Lett., vol.159, p.105075, 2022.

[11] S.-Y. Oh and H.-L. Choi, "Asymptotic stabilization of a chain of integrators by an event-triggered gain-scaling controller," IEICE Trans. Fundamentals, vol.E104-A, no.10, pp.1421–1424, Oct. 2021.

[12] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," IEEE Trans. Autom. Control, vol.52, no.9, pp.1680–1685, 2007.

[13] C.-H. Zhang and G.-H. Yang, "Event-triggered global finite-time control for a class of uncertain nonlinear systems," IEEE Trans. Autom. Control, vol.65, no.3, pp.1340–1347, 2020.