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The Institute of Electronics, Information and Communication Engineers
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PAPER

Game Theoretic Power Allocation and Antenna Selection for Target Detection in Hybrid Active and Passive MIMO Radar

Zhen WANG[†a)] ***and*** **Longye WANG**[††b)], ***Members***

SUMMARY In this paper, a hybrid active and passive (HAP) multiple-input multiple-output (MIMO) radar network is considered, where target returns from both active radar transmitters and illuminators of opportunity (IOs) are employed to complete target detection. With consideration for the active radar power limitation and the total available number constraint for the IOs, the joint discrete power allocation and antenna selection for target detection in HAP MIMO radar is studied. A game-theoretic framework is proposed to solve the problem where the target probability of detection (PD) of the HAP MIMO radar is utilized to build a common utility. The formulated discrete game is proven to be a potential game that possesses at least one pure strategy Nash equilibrium (NE) and an optimal strategy profile that maximizes the PD of HAP radar. The properties of the formulated game, including the feasibility, existence and optimality of NE, are also analyzed. The proposed game's pure strategy NE is determined to be an optimal scheme under certain conditions. An iterative algorithm is then designed to achieve the pure strategy NE. The designed algorithm's convergence and complexity are discussed. It is demonstrated that the designed algorithm can achieve almost optimal target detection performance while maintaining low complexity. Under certain conditions, the designed algorithm can obtain optimal performance.

key words: *Discrete power allocation, Antenna selection, Illuminators of opportunity (IOs), Target detection, Potential game.*

1. Introduction

With the update of electronic information technology, the electromagnetic environment becomes more and more complicated, which brings new challenges and opportunities to the traditional radar system and signal processing. For example, the commercialization of 5G communication technology has accelerated recently, resulting in amount of wireless communication equipment has expanded dramatically [1]. On the other hand, the future society tends to have everything connected to the intelligent internet of things [2], which may lead to a sharp rise in the number of wireless devices and a more complex electromagnetic environment. Thus, radar systems often coexist with other illuminators of opportunity (IOs). Since the signals reflected by these IOs contains useful target information, it can be exploited to complete radar tasks, which is called a passive radar system [3], [4]. These IOs, together with the original arrangement of active transmitters and receivers, form a hybrid active and passive (HAP) radar system. The HAP radar combines the advantages of active radar and passive radar, and has more advantages in fulfilling radar tasks. In addition, in many HAP systems mentioned above, there are usually a large number of receivers and transmitters, which are often widely distributed in the environment, forming a multiple-input multiple-output (MIMO) system[5]. Targets can be detected from multiple directions to enhance the detection performance for MIMO radar. Therefore, this paper focuses on HAP MIMO radar systems.

The HAP radar has more potential than conventional radar systems since it can take advantage of IOs in the surveillance region to assist radar tasks. Some work on coexisting radar and communication systems [6]–[12] has reflected the idea of HAP processing. The received communication signals reflected by the target have been used in [6]–[8] to enhance radar detection performance and parameter estimation. It has been demonstrated that an orthogonal frequency division multiplexing (OFDM) radar system can lower transmitted power and increase radar resolution by taking advantage of reflected communication signals [9]. The authors in [10] have shown that target location performance can be greatly improved by using HAP processing. The generalized maximum likelihood ratio detector has been employed in [11], indicating that the assist of communication signals provides significant gains in radar detection performance. Using communication signals reflected by vehicles, the authors in [12] have demonstrated enhanced detection performance in an automotive scenario.

In view of the above literatures verifying that the HAP processing can greatly improve radar performance, some work have studied the HAP radar in recent years [13]–[16]. In [13], the joint transmit-receive beamforming for HAP radar has been considered to maximize the signal-to-interference-plus-noise ratio. The design for joint optimization of the HAP radar's receive filters and radar waveform has been taken into consideration with timing uncertainty in [14]. In distributed HAP radar networks, a power allocation technique for target localization is proposed in [15] to optimize the power allocation of each active radar in view of power constraints. In [16], the resource optimization strategy was considered for HAP radar network engaged in multiple target tracking, which can optimize the received beams of both active and passive radars and the active radars' transmit power. None of the work mentioned above considered the joint power allocation of active radar and antenna selection of IOs in HAP MIMO radar. Moreover, the power allocation for the active part in [15], [16] assumed an ideal scenario where the power level may be adjusted continually. In practice, power levels are usually preferred as a series of discrete values [17]–[19]. For the passive part, the maximum number of IOs that the system can handle is usually

[†]The author is with National Key Laboratory of Wireless Communications, University of Electronic Science and Technology of China (UESTC), Chengdu, Sichuan 611731 China, School of Electrical Engineering and Information, Southwest Petroleum University, Chengdu, Sichuan 610500 China, and Yangtze Delta Region Institute Quzhou, UESTC, Quzhou, Zhejiang 324000 China. The work of Z. Wang was supported by the National Nature Science Foundation of China under Grant No. 62301465, and by the Natural Science Starting Project of SWPU (No.2022QHZ019). The work of L. Wang was supported by the National Natural Science Foundation of China under Grant No.62161047
a) E-mail: zhenwang0819@foxmail.com(Corresponding author)
b) E-mail: utibetwly@qq.com

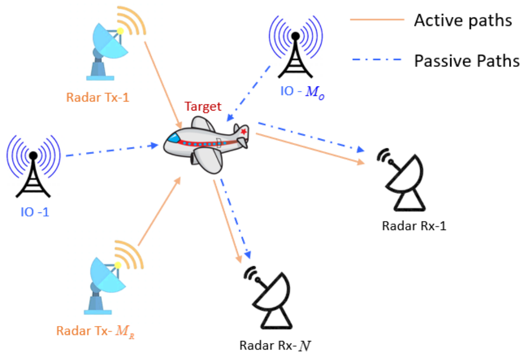


Fig. 1: HAP MIMO radar system.

limited, while there is a large amount of IOs available in the environment and the system's processing capacity is limited. In order to optimize system performance while considering the limitations of the system's processing capability, it is crucial to choose IOs reasonably. Therefore, it is crucial to study the joint discrete power allocation of active transmitters and antenna selection of IOs for the HAP radar system so that the system performance can be maximized with limited resources.

In this paper, the joint discrete power allocation of active transmitters and antenna selection of IOs is investigated in HAP radar system to maximize the target probability of detection (PD). A low-complex joint discrete power allocation and antenna selection scheme needs to be designed for HAP system under the constraints of the total number of IOs available and active radar power. An exhaustive search algorithm may usually solve the discrete optimization problem. However, because the exhaustive search algorithm's complexity increases exponentially as the number of active and passive transmitters, it becomes extremely impractical as the number of transmitters increases. Game theory, specifically potential games, is an appropriate tool that many studies have employed to address discrete optimization problems [17], [18], [20], [21]. This paper also adopts the potential game to solve the discrete optimization. First, the PD of the HAP radar is obtained, based on which a constrained discrete optimization problem is proposed. Next, a game-theoretic framework is employed to address the optimization problem with lower computational complexity where the PD is the common utility. The proposed game's properties is analyzed, and an iterative algorithm is developed to obtain a feasible pure strategy NE. Finally, the correctness of theoretical analysis is verified by simulation.

The subsequent sections of this work are structured as follows. The HAP MIMO radar network system model is introduced in Section II. In Section III, the target detection and problem formulation is presented. In Section IV, the proposed game' properties is investigated and the algorithm is designed to achieve the feasible pure strategy NE. Performance analysis is presented in Section VI. Finally, conclusions are drawn in Section VII.

2. Signal Model

The HAP MIMO radar system consists of M_R active radar transmitters, N receivers, and M_O IOs, all widely spaced, each using a single antenna, as depicted in Fig. 1. In a two-dimensional Cartesian coordinate system, the m_Rth, $m_R = 1, \ldots, M_R$ active radar transmitter are positioned at $(x^t_{R,m_R}, y^t_{R,m_R})$, the m_Oth, $m_O = 1, \ldots, M_O$ IO at $(x^t_{O,m_O}, y^t_{O,m_O})$, and the nth, $n = 1, \ldots, N$ radar receiver at (x^r_n, y^r_n). The m_Rth radar transmit signal is denoted as $\sqrt{P_{R,m_R}} s_{R,m_R}(kT_s)$, where T_s represents the sampling period, P_{R,m_R} is the radar transmit power, k $(k = 1, \ldots, K)$ is different indicators on the sample time, $K = \lceil T f_s \rceil$, the notation $\lceil \cdot \rceil$ denotes rounding up, f_s is the sampling rate, and T represents the observation interval. The signal emitted by the m_Oth IO is $\sqrt{P_{O,m_O}} s_{O,m_O}(kT_s)$, where P_{O,m_O} is the transmit power. Assuming a target is present at (x, y), the nth radar receiver's received signal at time kT_s can be expressed as

$$\begin{aligned} r_n[k] = &\sum_{m_R=1}^{M_R} \sqrt{\frac{P_{R,m_R} A_R}{d^2_{Rt,m_R} d^2_{r,n}}} \zeta_{R,nm_R} s_{R,m_R}(kT_s - \tau_{R,nm_R}) \\ &+ \sum_{m_O=1}^{M_O} \sqrt{\frac{P_{O,m_O} A_O}{d^2_{Ot,m_O} d^2_{r,n}}} \zeta_{O,nm_O} s_{O,m_O}(kT_s - \tau_{O,nm_O}) + w_n[k]. \end{aligned} \tag{1}$$

In (1), the first term is derived from the active radar transmitters, while the second term is derived from the IOs. The $d_{Rt,m_R} = [(x^t_{R,m_R} - x)^2 + (y^t_{R,m_R} - y)^2]^{1/2}$ is the distance between the m_Rth active transmitter and the target, $d_{r,n} = [(x^r_n - x)^2 + (y^r_n - y)^2]^{1/2}$ is the distance between the nth receiver and the target, $d_{Ot,m_O} = [(x^t_{O,m_O} - x)^2 + (y^t_{O,m_O} - y)^2]^{1/2}$ is the distance between the m_Oth passive transmitter and the target, A_R denotes the ratio of received power to transmitted active power at $d_{Rt,m} = d_{r,n} = 1$, and A_O denotes the ratio of received power to transmitted passive power at $d_{Ot,m} = d_{r,n} = 1$. The ζ_{R,nm_R} and ζ_{O,nm_O} represent the target reflection coefficients associated with the nm_Rth active and nm_Oth passive propagation paths, τ_{R,nm_R} and τ_{O,nm_O} are the corresponding time delays, f_{R,nm_R} and f_{O,nm_O} denote the corresponding Doppler frequencies, and $w_n[k]$ represents the clutter-plus-noise which is supposed to follow a white complex Gaussian distribution with zero-mean and $\mathbb{E}\{w_n[k]w^*_n[k']\} = \sigma^2_w \delta(k - k')$, where $\mathbb{E}[\cdot]$ stands for the mathematical expectation.

The m_Rth active radar transmitter's transmit power P_{R,m_R} is assumed to be chosen from a finite set $\mathcal{P}_{m_R}$, where $\mathcal{P}_{m_R} = \{P_{m_R 1}, P_{m_R 2}, \cdots, P_{m_R L_{m_R}}\}$, and L_{m_R} is the number of elements in the set of $\mathcal{P}_{m_R}$. The transmission strategy for active radar, in particular, comprises discrete power allocation and antenna selection in the HAP radar network where $P_{R,m_R} = 0$ is included. The m_Rth active radar transmit antenna is not chosen for the HAP radar network if $P_{R,m_R} = 0$. For passive part, it is possible to select which IO signal to be used on the receiver. The selection variable of the IO is defined as b_{m_O}, which is chosen from a finite set $\mathcal{B}_{m_O} = \{0, 1\}$, where 1 indicates that the IO is selected, and 0 indicates that the IO is not selected. Consequently, the signal model (1) can be further written as

$$\begin{aligned} r_n[k] = &\sum_{m_R=1}^{M_R} \sqrt{\frac{P_{R,m_R} A_R}{d^2_{Rt,m_R} d^2_{r,n}}} \zeta_{R,nm_R} s_{R,m_R}(kT_s - \tau_{R,nm_R}) \\ &+ \sum_{m_O=1}^{M_O} \sqrt{\frac{P_{O,m_O} A_O}{d^2_{Ot,m_O} d^2_{r,n}}} b_{m_O} \zeta_{O,nm_O} s_{O,m_O}(kT_s - \tau_{O,nm_O}) + w_n[k], \end{aligned} \tag{2}$$

where $P_{R,m_R} \in \mathcal{P}_{m_R} = \{P_{m_R 1}, P_{m_R 2}, \cdots, P_{m_R L_{m_R}}\}$, and

$b_{m_O} \in \mathcal{B}_{m_O} = \{0, 1\}$.

The received signal vector can be expressed as

$$\mathbf{r}_n = \left[r_n[1], \cdots, r_n[K]\right]^{\dagger} = \boldsymbol{\mu}_{R,n} + \boldsymbol{\mu}_{O,n} + \mathbf{w}_n, \quad (3)$$

where the superscript "†" stands for transpose, $\mathbf{w}_n = \left[w_n[1] \cdots, w_n[K]\right]^{\dagger}$, $\boldsymbol{\mu}_{R,n} = \left[\mu_{R,n}[1], \cdots, \mu_{R,n}[K]\right]^{\dagger}$, and $\boldsymbol{\mu}_{O,n} = \left[\mu_{O,n}[1], \cdots, \mu_{O,n}[K]\right]^{\dagger}$, in which

$$\mu_{R,n}[k] = \sum_{m_R=1}^{M_R} \sqrt{\frac{P_{R,m_R} A_R}{d_{Rt,m_R}^2 d_{r,n}^2}} \zeta_{R,nm_R} s_{R,m_R}(kT_s - \tau_{R,nm_R}),$$
$$\mu_{O,n}[k] = \sum_{m_O=1}^{M_O} \sqrt{\frac{P_{O,m_O} A_O}{d_{Ot,m_O}^2 d_{r,n}^2}} b_{m_O} \zeta_{O,nm_O} s_{O,m_O}(kT_s - \tau_{O,nm_O}).$$

The overall received signal when the target is present can be written as

$$\mathbf{r} = \left[\mathbf{r}_1^{\dagger}, \mathbf{r}_2^{\dagger}, \cdots, \mathbf{r}_N^{\dagger}\right]^{\dagger} = \boldsymbol{\mu}_R + \boldsymbol{\mu}_O + \mathbf{w}, \quad (4)$$

in which $\boldsymbol{\mu}_R = [\boldsymbol{\mu}_{R,1}^{\dagger}, \cdots, \boldsymbol{\mu}_{R,N}^{\dagger}]^{\dagger}$, $\boldsymbol{\mu}_O = [\boldsymbol{\mu}_{O,1}^{\dagger}, \cdots, \boldsymbol{\mu}_{O,N}^{\dagger}]^{\dagger}$ and $\mathbf{w} = [\mathbf{w}_1^{\dagger}, \cdots, \mathbf{w}_N^{\dagger}]^{\dagger}$.

3. Target Detection and Problem Formulation

This section first discusses the target detection of the HAP radar system, and then analyzes the maximization of the probability of detection, based on which a game-theoretic framework is employed to solve the optimization problem.

3.1 Target Detection

The hypotheses are represented by H_0 for target absent and H_1 for target present. According to the received signal model described in (4), the detection problem can be formulated as follows

$$\begin{aligned} H_0 &: \mathbf{r} = \mathbf{w}, \\ H_1 &: \mathbf{r} = \boldsymbol{\mu}_R + \boldsymbol{\mu}_O + \mathbf{w}, \end{aligned} \quad (5)$$

where $\mathbf{r}|H_0 \sim \mathcal{CN}\left(\mathbf{0}, \sigma_w^2 \boldsymbol{I}\right)$, $\mathbf{r}|H_1 \sim \mathcal{CN}\left(\boldsymbol{\mu}_R + \boldsymbol{\mu}_O, \sigma_w^2 \boldsymbol{I}\right)$, and $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{C})$ denotes a complex Gaussian distribution characterised by a mean vector $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{C}$. Consequently, the log-likelihood ratio can be obtained

$$\begin{aligned} L(\mathbf{r}) &= \log \tfrac{f(\mathbf{r}|H_1)}{f(\mathbf{r}|H_0)}, \\ &= \left\{2\Re\left\{\mathbf{r}^{\mathbf{H}}(\boldsymbol{\mu}_{\mathbf{R}} + \boldsymbol{\mu}_{\mathbf{O}})\right\} - |\boldsymbol{\mu}_R + \boldsymbol{\mu}_O|^2\right\} / \sigma_w^2, \end{aligned} \quad (6)$$

where $f(\mathbf{r}|H_1)$ and $f(\mathbf{r}|H_0)$ represent the probability density functions of the observation vector $\mathbf{r}$ under the two hypotheses. Including these terms which are independent of $\mathbf{r}$ in the test threshold, the test statistic can be obtained as

$$\mathbb{T}(\mathbf{r}) = \Re\left\{\mathbf{r}^{\mathbf{H}}(\boldsymbol{\mu}_{\mathbf{R}} + \boldsymbol{\mu}_{\mathbf{O}})\right\}, \quad (7)$$

where $\mathbb{T}(\mathbf{r})|H_0 \sim \mathcal{N}(0, \sigma^2)$, $\mathbb{T}(\mathbf{r})|H_1 \sim \mathcal{N}(\mu_1, \sigma^2)$, $\mu_1 = |\boldsymbol{\mu}_R + \boldsymbol{\mu}_O|^2$, and $\sigma^2 = \sigma_w^2 |\boldsymbol{\mu}_R + \boldsymbol{\mu}_O|^2/2$.

Utilizing the Neyman-Pearson (NP) criterion, the optimal detector is determined by [22]

$$\mathbb{T}(\mathbf{r}) = \Re\left\{\mathbf{r}^{\mathbf{H}}(\boldsymbol{\mu}_{\mathbf{R}} + \boldsymbol{\mu}_{\mathbf{O}})\right\} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \quad (8)$$

where η denotes the detection threshold that is decided by the desired false alarm probability $P_{FA} = Pr(\mathbb{T}(\mathbf{r}) > \eta|H_0)$, and the $Pr(\mathbb{A}|H_i)$ represents the probability of event $\mathbb{A}$ under hypothesis H_i.

From (7), the false alarm probability is

$$P_{\text{FA}} = Pr(\mathbb{T}(\mathbf{r}) > \eta|H_0) = Q\left(\frac{\eta}{\sigma}\right), \quad (9)$$

and the detection threshold η is thereby $\eta = \sigma Q^{-1}(P_{\text{FA}})$, where $Q(\cdot)$ denotes the standard Gaussian distribution's complementary distribution function, which is expressed as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt . \quad (10)$$

Consequently, for the HAP radar system the radar target probability of detection can be calculated as

$$P_D = Pr(\mathbb{T}(\mathbf{r}) \geq \eta|H_1) = Q\left(Q^{-1}(P_{\text{FA}}) - \mu\right), \quad (11)$$

where $\mu = \frac{\mu_1}{\sigma} = \sqrt{2|\boldsymbol{\mu}_R + \boldsymbol{\mu}_O|^2}/\sigma_w$.

3.2 Probability of Detection Maximization Problem

When the total transmit power of the active radar and the number of IOs that the system can handle are limited, the probability of detection maximization problem for joint discrete power allocation and antenna selection in the HAP radar network can be described as

$$\begin{aligned} &\text{Problem 1}: \max_{P_{R,m_R} \in \mathcal{P}_{m_R}, b_{m_O} \in \mathcal{B}_{m_O}} P_D \\ &s.t. \begin{cases} \sum_{m_R=1}^{M_R} P_{R,m_R} \leq P_{total}, \ (C1) \\ \sum_{m_O=1}^{M_O} b_{m_O} \leq M_{total}, \ (C2) \end{cases} \end{aligned} \quad (12)$$

where $C1$ represents the total power constraint for the active radar and P_{total} is the total available power, and $C2$ denotes the maximum number of IOs that the system can process and M_{total} is the total available number of IOs.

For the convenience of subsequent presentation, define

$$P_m = \begin{cases} P_{R,m}, \ m = 1, \cdots, M_R \\ b_{m-M_R}, \ m = M_R + 1, \cdots, M_R + M_O \end{cases} \quad (13)$$

which are chosen from a finite sets

$$\mathcal{P}_m = \begin{cases} \mathcal{P}_{R,m} = \{P_{m1}, P_{m2}, \cdots, P_{mL_m}\}, \ m = 1, \cdots, M_R, \\ \mathcal{B}_{m-M_R} = \{0, 1\}, \ m = M_R + 1, \cdots, M_R + M_O. \end{cases} \quad (14)$$

Further, define $S = |\boldsymbol{\mu}_R + \boldsymbol{\mu}_O|^2 = \sum_{n=1}^{N} \sum_{k=1}^{K} \left|\mu_{R,n}[k] + \mu_{O,n}[k]\right|^2$, which can be expressed as

$$\begin{aligned} S = &\sum_{m_R=1}^{M_R} \sum_{m_R'=1}^{M_R} \sqrt{P_{R,m_R} P_{R,m_R'}} \alpha_{m_R m_R'} + \sum_{m_O=1}^{M_O} \sum_{m_O'=1}^{M_O} \sqrt{b_{m_O} b_{m_O'}} \gamma_{m_O m_O'} \\ &+ 2 \sum_{m_R=1}^{M_R} \sum_{m_O=1}^{M_O} \sqrt{P_{R,m_R} b_{m_O}} \kappa_{m_R m_O}, \end{aligned} \quad (15)$$

in which

$$\alpha_{m_R m'_R} = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{A_R}{d^2_{Rt,m_R} d^2_{r,n}} \zeta_{R,nm_R} \zeta^*_{R,nm'_R} s_{R,m_R}(kT_s - \tau_{R,nm_R}) \times s^*_{R,m'_R}(kT_s - \tau_{R,nm'_R}),$$

$$\kappa_{m_R m_O} = \sum_{n=1}^{N}\sum_{k=1}^{K} \Re\left\{\sqrt{\frac{P_{O,m_O} A_O A_R}{d^2_{Ot,m_O} d^2_{Rt,m_R} d^4_{r,n}}} \zeta_{R,nm_R} \zeta^*_{O,nm_O} \times s_{R,m_R}(kT_s - \tau_{R,nm_R}) s^*_{O,m_O}(kT_s - \tau_{O,nm_O})\right\},$$

$$\gamma_{m_O m'_O} = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{P_{O,m_O} A_O}{d^2_{Ot,m_O} d^2_{r,n}} \zeta_{O,nm_O} \zeta^*_{O,nm'_O} \times s_{O,m_O}(kT_s - \tau_{O,nm_O}) s^*_{O,m'_O}(kT_s - \tau_{O,nm'_O}).$$

From (13) and (15), $S = \sum_{n=1}^{N}\sum_{k=1}^{K} \left|\mu_{R,n}[k] + \mu_{O,n}[k]\right|^2$ can be written as

$$S = \sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m P_{m'}} \rho_{mm'}, \tag{16}$$

in which

$$\rho_{mm'} = \begin{cases} \alpha_{mm'}, & m, m' = 1, \cdots, M_R, \\ \kappa_{m(m'-M_R)}, & m = 1, \cdots, M_R, m' = M_R + 1, \cdots, M_R + M_O, \\ \kappa_{m'(m-M_R)}, & m = M_R + 1, \cdots, M_R + M_O, m' = 1, \cdots, M_R, \\ \gamma_{(m-M_R)(m'-M_R)}, & m, m' = M_R + 1, \cdots, M_R + M_O. \end{cases} \tag{17}$$

The PD in (11) increases monotonically with the increase of $\mu = \mu_1/\sigma$, which is determined by S in (16), as the Gaussian complementary function in (10) is a monotonically decreasing function. Thus, the Problem 1 can be recast as

$$\text{Problem 2}: \max_{P_m \in \mathcal{P}_m} S = \sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m P_{m'}} \rho_{mm'}$$
$$s.t. \begin{cases} \sum_{m=1}^{M_R} P_m \le P_{total}, \ (C1) \\ \sum_{m=M_R+1}^{M_R+M_O} P_m \le M_{total}. \ (C2) \end{cases} \tag{18}$$

3.3 Game-Theoretic Formulation

Problem 2 is undoubtedly a discrete optimization problem with constraints that can be solved directly using exhaustive search algorithms. Nevertheless, an exhaustive search algorithm (ESA) is highly inappropriate, especially for large systems, since it has to search $\mathbb{O}(2^{M_O} \prod_{m=1}^{M_R} L_m)$ antenna selection and power allocation strategies and compare their corresponding performance. For example, supposing that each active transmitter in a HAP radar has five optional power strategies and that $M_O = 10, M_R = 6$, the ESA needs to search all $2^{10}5^6 = 16000000$ combinations of power allocation of active transmitters and antenna selection of IOs. In such case, the ESA has a high computational complexity and requires a significant amount of memory.

Achieving the optimal scheme of Problem 2 is a challenging task. Game theory offers a promising alternative to this issue. The joint active power allocation and passive antenna selection problem is addressed using a game-theoretic approach, specifically applying potential games to represent the incentives of all players through a global function [20].

All active and passive radar transmitters are taken into consideration here with the same utility. Furthermore, no transmitter is allowed to utilize a transmission strategy profile that is in violation of either the $C1$ or $C2$. This is challenging because it is hard to determine beforehand which strategy profiles are infeasible and which are feasible for all transmitters. Consequently, S in (18) is not suitable to be applied directly as the utility function. Referring to (18), the common utility function of transmitter m $(m = 1, 2, \cdots, M_R, \cdots, M_R + M_O)$ is defined as [17]

$$U = S + \beta^P \Theta(P_{total} - \sum_{m=1}^{M_R} P_m) + \beta^M \Theta\left(M_{total} - \sum_{m=M_R+1}^{M_R+M_O} P_m\right), \tag{19}$$

with β^P and β^M being non-negative scalars, and the penalty function $\Theta(s)$ being

$$\Theta(s) = \begin{cases} s, \ if \ s < 0, \\ 0, \ otherwise. \end{cases} \tag{20}$$

The second term in (19) stands for the total power constraint for the active transmission, whereas the third term in (19) represents the constraint on maximum number of IOs for the passive transmission.

Then the joint power allocation and antenna selection for the HAP radar can be defined as a discrete game

$$\mathcal{G} = \left[\mathcal{K}, \{\mathcal{P}_i\}_{i\in\mathcal{K}}, \{u_i\}_{i\in\mathcal{K}}\right], \tag{21}$$

in which $\mathcal{K}=\{1, 2, \cdots, M_R, \cdots, M_R + M_O\}$ is the set player, including the index set for all passive and active transmitters in the HAP system, $\mathcal{P}_i$ is the pure strategy set of the ith transmitter, and u_i is the utility function of the ith transmitter. Therefore, it can be seen that the proposed game is a common payoff game and the utility function U is specified in (19). In other words, $u_i = u_j = U, \forall i, j \in \mathcal{K}$. As a result, $\mathcal{G}$ is a potential game in which U is the potential function [20].

4. Properties of Proposed Game

This section analyze the feasibility and existence of the proposed game's Nash equilibrium (NE). Subsequently, the optimality of NE, which is the relationship between the optimal scheme and the NE to Problem 2, is examined.

Definition 1. (Pure strategy NE) [23] A strategy profile $(P_1^*, \cdots, P^*_{M_R+M_O})$ is a pure strategy NE of $\mathcal{G}$, if $\forall i \in \mathcal{K}$ and an alternate strategy $P_i \neq P_i^*, P_i \in \mathcal{P}_i$, we have

$$u_i\left(P_i^*, P_{-i}^*\right) \ge u_i\left(P_i, P_{-i}^*\right). \tag{22}$$

4.1 Feasibility

If no constraints exist, the proposed game's NE can serve as a solution to Problem 2. Determining if the NE satisfies $C1$ and $C2$ is difficult when the HAP system is constrained by them. Following this, the feasibility of the NE is examined.

Define

$$\beta^P_{th} = \max\left\{\max\left[\frac{S(P_i, P_{-i})}{P_i}\right], \max\left[\frac{S(P_i, P_{-i})}{\sum_{j\in\mathcal{K}_{M_R}} P_j - P_{total}}\right]\right\} \quad i \in \mathcal{K}_{M_R}, P_i \in \mathcal{P}_i, P_i \neq 0,$$
$$\beta^M_{th} = \max\left\{\max\left[\frac{S(P_i, P_{-i})}{P_i}\right], \max\left[\frac{S(P_i, P_{-i})}{\sum_{j\in\mathcal{K}-\mathcal{K}_{M_R}} P_j - M_{total}}\right]\right\} \quad i \in \mathcal{K} - \mathcal{K}_{M_R}, P_i \in \mathcal{P}_i, P_i \neq 0, \tag{23}$$

where $\mathcal{K}_{M_R} = \{1, 2, \cdots, M_R\}$ denotes the set of the first M_R players of $\mathcal{K}$, and $\mathcal{K} - \mathcal{K}_{M_R}$ represents the set of $\mathcal{K}$ except $\mathcal{K}_{M_R}$.

Theorem 1. The pure strategy NE of game $\mathcal{G}$ must be feasible if $\beta^P > \beta^P_{th}$ and $\beta^M > \beta^M_{th}$, .

Proof: See Appendix Appendix A. □

4.2 Existence

In addition to feasibility, another essential matter to explore in game-theoretic networks is the existence of pure strategy NE. A game may not always have a pure strategy NE, however the proposed game $\mathcal{G}$ is a game with common utility, which allows for obtaining the following results.

Theorem 2. A maximizer of U in (19), that is an optimal scheme to Problem 2, is a feasible pure strategy NE of game $\mathcal{G}$ if $\beta^P > \beta^P_{th}$ and $\beta^M > \beta^M_{th}$.

Proof: Suppose that $(P^o_1, \cdots, P^o_{M_R+M_O})$ maximizes U in (19).It is clear that $U(P^o_1, \cdots, P^o_{M_R+M_O}) \geq 0$. If $(P^o_1, \cdots, P^o_{M_R+M_O})$ is not feasible, then $U(P^o_1, \cdots, P^o_{M_R+M_O}) < 0$ contradicting the previous inequality. Hence, if the theorem's conditions are satisfied, $(P^o_1, \cdots, P^o_{M_R+M_O})$ is feasible, resulting in $U(P^o_1, \cdots, P^o_{M_R+M_O}) = S(P^o_1, \cdots, P^o_{M_R+M_O})$. Since $(P^o_1, \cdots, P^o_{M_R+M_O})$ can maximize U, for any feasible strategy profile $(P_1, \cdots, P_{M_R+M_O})$, we have $S(P^o_1, \cdots, P^o_{M_R+M_O}) \geq S(P_1, \cdots, P_{M_R+M_O})$, when the theorem's conditions are satisfied. Consequently, $(P^o_1, \cdots, P^o_{M_R+M_O})$ is also an optimal scheme to Problem 2.

For every i in set $\mathcal{K}$, let P_i represent an alternate strategy of player i, where $P_i \in \mathcal{P}_i$ and $P_i \neq P^o_i$. Since $u_i = u_j = U$, $\forall i, j \in \mathcal{K}$, it follows that

$$u_i\left(P^o_i, P^o_{-i}\right) \geq u_i\left(P_i, P^o_{-i}\right). \tag{24}$$

As a result of this, it is impossible for any player to unilaterally alter their strategy to enhance its utility, so $(P^o_1, \cdots, P^o_{M_R+M_O})$ is a pure strategy NE as defined in Definition 1, which completes the proof. □

4.3 Optimality

Nevertheless, understanding the quality of the NE is also crucial. The relationship between the exhaustive search algorithm-obtained optimal scheme to Problem 2 and the NE of game $\mathcal{G}$ is studied in this subsection.

Theorem 2 shows that the optimal scheme to Problem 2 is also a feasible pure strategy NE of $\mathcal{G}$ if $\beta^P > \beta^P_{th}$ and $\beta^M > \beta^M_{th}$. Hence, if the game $\mathcal{G}$ has a unique NE, the NE is the optimal scheme. However, determining the number of NEs for game $\mathcal{G}$ is generally challenging. When the game $\mathcal{G}$ has multiple NEs, it is important to note that the NE may not always be the optimal scheme, as per the definition of NE. Fortunately, for the joint discrete optimization problem in this paper, when the HAP system meets certain conditions, the NE of game $\mathcal{G}$ and the optimal scheme are equivalent, which is described in Theorem 3 below.

Theorem 3. If all of the following conditions are satisfied, then a pure strategy NE of game $\mathcal{G}$ is the optimal scheme to Problem 2:

(i) all strategy profiles are guaranteed to satisfy $C1$ and $C2$;

(ii) $\rho_{mm'} \geq 0, \forall m, m' \in \{1, \cdots, M_R + M_O\}$ in (17).

Proof: See Appendix Appendix B. □

Note that by appropriately selecting penalty factors β^P and β^M, the condition (i) of Theorem 3 can be satisfied. In practical systems, it is common for all active and passive transmitted signals to be orthogonal to one another, ensuring the Theorem 3's condition (ii). In such scenarios, it is always possible to simply solve Problem 2 by obtaining a pure strategy NE of game $\mathcal{G}$.

5. Joint Power Allocation and Antenna Selection Algorithm

Theorems 1-3 demonstrate that there is a feasible pure strategy NE for the proposed game given certain conditions. Moreover, if the conditions of Theorem 3 are satisfied, a pure strategy NE is an optimal scheme for the joint power allocation and antenna selection in HAP radar. This indicates that, in some cases, a local search algorithm, instead of the complicated exhaustive search, can provide an optimal scheme to Problem 2. This interesting finding can guide the design of practical systems.

Consequently, a low-complex iterative algorithm is designed to get a pure strategy NE, which is called the iterative power allocation and antenna selection algorithm (IPAASA) and presented as Algorithm 1.

Algorithm 1 IPAASA

Initialize the strategy profile $\left(P^0_i, P^0_{-i}\right)$ and set $t = 0$;
repeat
  for $i = 1 : M_R + M_O$
    $P^{t+1}_i = \arg\max_{P_i \in \mathcal{P}_i} u_i\left(P_i, P^t_{-i}\right)$;
    $P^t_i = P^{t+1}_i$;
  end
  Update $t = t + 1$;
until $u_i\left(P^t_i, P^t_{-i}\right) = u_i\left(P^{t-1}_i, P^{t-1}_{-i}\right), \forall i \in \mathcal{K}$
return $\left(P^t_i, P^t_{-i}\right)$;

When Algorithm 1 converges to a strategy profile $(P^*_1, \cdots, P^*_{M_R+M_O})$, we have

$$u_i\left(P^*_i, P^*_{-i}\right) \geq u_i\left(P'_i, P^*_{-i}\right)\ , \forall i \in \mathcal{K}, \tag{25}$$

where P'_i is any alternate pure strategy that is not equal to P^*_i. From Definition 1, $(P^*_1, \cdots, P^*_{M_R+M_O})$ is a pure strategy NE of $\mathcal{G}$. In addition, based on Theorem 1, $(P^*_1, \cdots, P^*_{M_R+M_O})$ is feasible if $\beta^P > \beta^P_{th}$ and $\beta^M > \beta^M_{th}$. This means that Algorithm 1 converges to a feasible pure strategy NE of game $\mathcal{G}$.

The thresholds of the penalty factors are set sufficiently enough in Algorithm 1 such that $\beta^P > \beta^P_{th}$ and $\beta^M > \beta^M_{th}$. Since they are merely boundaries of penalty factors, knowing their precise values is not necessary in practice. To guarantee that the requirements of Theorems 1, 2, and 3 are satisfied, the penalty factors can be any positive real integers greater than β^P_{th} and β^M_{th}. It is also possible to dynamically modify the penalty factors at each iteration of Algorithm 1 so that the conditions of Theorems 1, 2, and 3 are satisfied. In each iteration t, the values of $\beta^P_{th}(t)$ and $\beta^M_{th}(t)$ are determined by referring to (23). If $\beta^P > \beta^P_{th}(t)$ and $\beta^M > \beta^M_{th}(t)$, β^P and β^M stay unaltered, otherwise, if $\beta^P < \beta^P_{th}(t)$, $\beta^P = \beta^P_{th}(t)$, and if $\beta^M < \beta^M(t)$, $\beta^M = \beta^M_{th}(t)$.

The complexity requirements of Algorithm 1 are then analyzed to obtain the pure strategy NE and compared with the ESA. Algorithm 1 searches for $O(t\{\sum_{m=1}^{M_R} L_m + 2M_O\})$ combinations with t denoting the total number of iterations, whereas the ESA searches all $O(2^{M_O}\prod_{m=1}^{M_R} L_m)$ strategy profiles. The complexity requirements of Algorithm 1 and the ESA increase significantly with the growing number of active and passive transmitters in the HAP radar system. In contrast to the exponential growth required by the ESA, the complexity of Algorithm 1 grows only linearly with the number of transmitters. As there are usually a big number of active transmitters and IOs in HAP radar system, Algorithm 1 can greatly reduce memory overhead compared with the ESA.

6. Performance Analysis

This section provides examples to demonstrate the detection performance of the HAP radar system. Assuming each active radar transmitter, receiver, and IO is positioned 7km away from the origin of the coordinate system, with a uniform dispersion in angle across the range $[0; 2\pi)$. The number of receivers is $N = 4$. For the active transmitters, the available transmit power set is $\mathcal{P}_{m_R} = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ $Watts$, $m_R \in 0, 1, \cdots, M_R$.

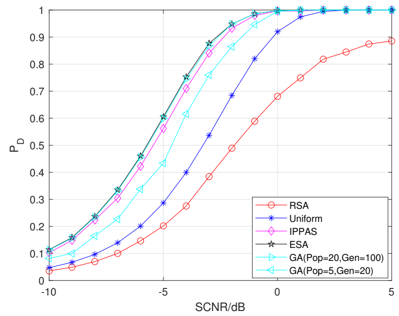
The waveforms transmitted by the active part are frequency spread single Gaussian pulse signals $s_{R,m_R}(t) = (2/T_R^2)^{(1/4)} \exp(-\pi t^2/T_R^2) e^{j2\pi m_R f_\Delta t}$, where f_Δ represents the frequency difference between adjacent radar transmit signals and T_R is the pulsewidth. Set $f_\Delta = 100Hz$ and $T_R = 0.01$. The OFDM signals are adopted for passive transmission $s_{O,m_O}(t) = \sum_{n=-N_f/2}^{N_f/2-1} a_{m_O}[n] e^{j2\pi n \Delta f t}$, where $a_{m_O}[n]$ denotes data symbols, Δf is the subcarrier frequency interval, and N_f is the number of subcarriers. Let $\Delta f = 333$ Hz, and $N_f = 6$. The sampling frequency is $f_s = 1250Hz$ and the observation interval is $T = 0.01s$. Define the signal to clutter-plus-noise ratio (SCNR) as

$$SCNR = 10\log_{10} \frac{\sum_{n=1}^{N} \left\{ \sum_{m=1}^{M_R} \frac{A_R}{d^2_{Rt,m_R} d^2_{r,n}} + \sum_{m=1}^{M_O} \frac{p_{O,m_O} A_O}{d^2_{Ot,m_O} d^2_{r,n}} \right\}}{N\sigma_w^2}. \quad (26)$$

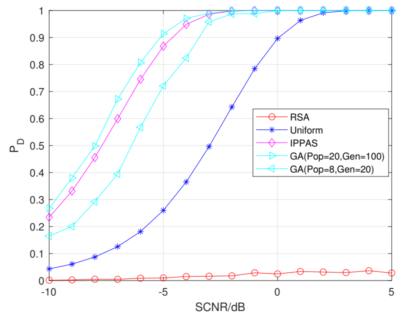
Assume that there is a target located at (2500, 2300)m and the probability of false alarm is set at a fixed value $P_{FA} = 10^{-3}$.

The proposed game is evaluated in comparison with an exhaustive search algorithm (ESA). A commonly used discrete optimization algorithm, the genetic algorithm (GA) [24], is also provided for comparison, showing the performance of different population size (Pop) and generations (Gen). Additionally, the random selection algorithm (RSA), in which each transmitter randomly selects its transmission strategy, is also presented for comparison over 1000 independent realizations. In RSA, when the chosen transmission strategy profile does not satisfy either one or both of C1 and C2, the PD is adjusted to 0. Moreover, the uniform scheme (Uniform) which means that all active radar transmitters allocate the available power uniformly is also provided with constraint C1.

Fig.2 provides the PDs versus SCNR for ESA, RSA, GA, uniform and the proposed IPAASA for various number of active transmitters with constraint C1, that is, the active radar power allocation problem is considered. In this case, the total



(a) $M_R = 6$



(b) $M_R = 12$

Fig. 2: P_D versus SCNR of RSA, GA, Uniform, IPAASA and ESA for HAP MIMO radar with $C1$ and $P_{total} = 4Watts$

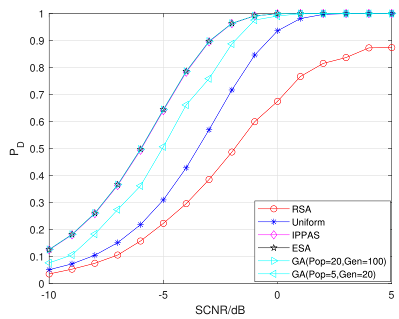


Fig. 3: P_D versus SCNR of RSA, GA, Uniform and IPAASA for HAP MIMO radar with $C1$, $M_R = 6$, $P_{total} = 4Watts$ and with non-uniform distributed transmitters and receivers.

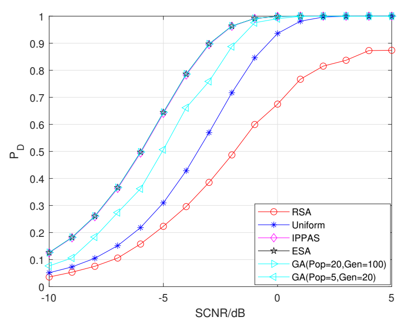


Fig. 4: P_D versus SCNR of RSA, GA, Uniform and IPAASA for HAP MIMO radar with $C1$, $M_R = 6$, $P_{total} = 4Watts$ and orthogonal transmitted signals.

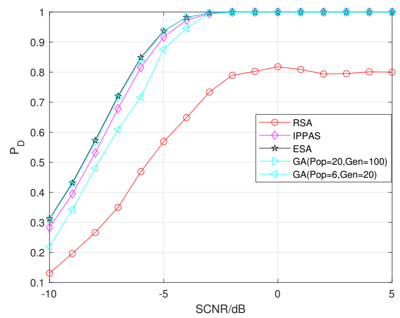


Fig. 5: P_D versus SCNR of RSA, GA, ESA and IPAASA for HAP MIMO radar with $C2$, $M_O = 12$, $M_{total} = 8$

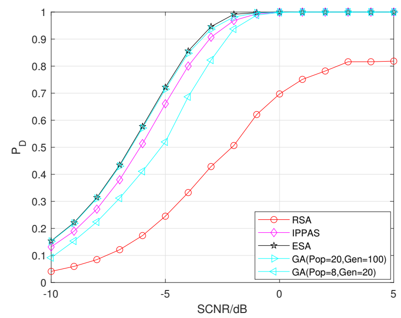


Fig. 6: P_D versus SCNR of RSA, GA, ESA and IPAASA for HAP MIMO radar with $C1$, $C2$, $M_R = 4$,$M_O = 6$, $P_{total} = 3Watts$, $M_{total} = 5$.

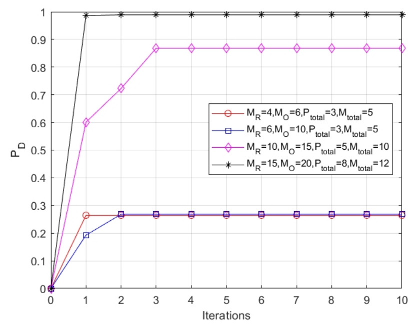


Fig. 7: Convergence rate of IPAASA under different numbers of M_R and M_{total} with $C1$, $C2$, and $SCNR = -3dB$.

power constraint for the active radar is $P_{total} = 4Watts$. In Fig.2 (b), ESA's performance is absent when $M_R = 12$ due to the high complexity. As can be seen from figures, the proposed IPAASA significantly outperforms RSA and uniform algorithm. Given that the complexity of the GA algorithm is $O(Pop \cdot Gen)$, when $Pop = 5, Gen = 20$, the complexity of the GA algorithm aligns with that of the proposed algorithm. When $Pop = 20, Gen = 100$, the complexity of the GA algorithm is significantly greater.This figure illustrates that the proposed algorithm outperforms the GA under similar complexity conditions. Moreover, the target detection performance of IPAASA is comparable to that of ESA.

In order to simplify the scenario, Fig.2 assumes that all transmitters and receivers are uniformly distributed in angle across the range $[0; 2\pi)$, while Fig.3 considers the scenario where all transmitters and receivers are not uniformly distributed when $M_R = 6$. Assume that all transmitters and receivers are randomly distributed over a range of 7 km. The results in Fig.3 are basically consistent with the results presented in Fig.2 (a), indicating that the proposed method is effective regardless of uniform distributed or non-uniform distributed transmitters and receivers. No special explanation is given later, all transmitters and receivers are assumed to be uniformly distributed.

In Fig. 4, the parameters are the same as those in Fig. 2 except that all the active transmitted signals satisfy orthogonality by setting $f_\Delta = 800Hz$ [25], that is , $\rho_{mm'} \geq 0, \forall m, m' \in \{1, \cdots, M_R + M_O\}$ in (17). The results depicted in Fig. 4 indicate that the proposed algorithm nearly matches the performance of ESA, thus confirming the validity of condition (ii) in Theorem 3.

Fig.5 provides the PDs versus SCNR for ESA, GA, RSA, and the proposed IPAASA with constraint C2 where $M_O = 12$ and $M_{total} = 8$, that is, the passive antenna selection problem is considered. As can be shown in the figure, the proposed IPAASA performs significantly better than the RSA and the GA under similar complexity conditions. Additionally, the IPAASA's target detection performance is comparable to that of ESA.

Fig.6 depicts the PDs versus SCNR for ESA, GA, RSA and the proposed IPAASA when constraints C1 and C2 are both taken into account. Set $M_R = 4$, $M_O = 6$, $P_{total} = 3Watts$, $M_{total} = 5$. According to the figure, the proposed approach consistently outperforms RSA and the GA under similar complexity conditions. Moreover, the the proposed IPAASA performs nearly as well as the ESA algorithm when it comes to joint power allocation and antenna selection in the HAP radar system.

Fig.7 displays the convergence rate of IPAASA for various numbers of active radar transmitters and IOs with $SCNR = -3dB$. The results show that the IPAASA illustrates a remarkably rapid convergence rate in the simulations, requiring less than 4 iterations. In addition, the convergence rate of IPAASA remains uninfluenced by the number of active radar transmitters and IOs. It can be concluded that the complexity of Algorithm 1 is significantly lower compared with the exhaustive search method $O(2^{M_O} \prod_{m=1}^{M_R} L_m)$.

7. Conclusions

In this paper, the joint discrete power allocation and antenna selection for target detection was addressed in HAP MIMO radar using game theory. The feasibility, existence and optimality of pure strategy NE in the proposed game have been demonstrated, based on which an iterative algorithm then is developed to obtain the pure strategy NE. It was demonstrated that the proposed algorithm can achieve near-optimal target detection performance while greatly decreasing the computational complexity in comparison with the optimal exhaustive search algorithm. Moreover, the designed algorithm can obtain optimal performance under certain conditions.

Appendix A: Proof of Theorem 1

Consider the strategy profile $(P_1^*, \cdots, P_{M_R+M_O}^*)$, which fails to meet $C1$ of player i and is a pure strategy NE of game $\mathcal{G}$. Then, $i \in \mathcal{K}_{M_R}$, $(P_1^*, \cdots, P_{M_R+M_O}^*) \neq (0, \cdots, 0)$, and $P_i^* \neq 0$. The utility function of player i can be written as

$$\begin{aligned}u_i\left(P_i^*, P_{-i}^*\right) &= S\left(P_i^*, P_{-i}^*\right) + \beta^P \Theta\left(P_{total} - \sum_{m=1}^{M_R} P_m^*\right)\\ &+\beta^M \Theta\left(M_{total} - \sum_{m=M_R+1}^{M_R+M_O} P_m^*\right),\\ u_i\left(0, P_{-i}^*\right) &= S\left(0, P_{-i}^*\right) + \beta^P \Theta\left(P_{total} - \sum_{m=1,m\neq i}^{M_R} P_m^*\right)\\ &+\beta^M \Theta\left(M_{total} - \sum_{m=M_R+1}^{M_R+M_O} P_m^*\right).\end{aligned} \tag{A· 1}$$

Given that $\beta^P > \beta_{th}^P$, if $P_{total} - \sum_{m=1,m\neq i}^{M_R} P_m^* \geq 0$, it can be obtained that

$$\begin{aligned}u_i\left(P_i^*, P_{-i}^*\right) - u_i\left(0, P_{-i}^*\right) &= S\left(P_i^*, P_{-i}^*\right) - S\left(0, P_{-i}^*\right)\\ &-\beta^P\left(\sum_{m=1}^{M_R} P_m^* - P_{total}\right) < 0,\end{aligned} \tag{A· 2}$$

and else if $P_{total} - \sum_{m=1,m\neq i}^{M_R} P_m^* < 0$, it can be obtained

$$u_i\left(P_i^*, P_{-i}^*\right) - u_i\left(0, P_{-i}^*\right) = S\left(P_i^*, P_{-i}^*\right) - S\left(0, P_{-i}^*\right) - \beta^P P_i^* < 0. \tag{A· 3}$$

Thus, from (A· 2) and (A· 3), it can be obtained that

$$u_i\left(P_i^*, P_{-i}^*\right) - u_i\left(0, P_{-i}^*\right) < 0, \tag{A· 4}$$

which goes against the assumption that $(P_1^*, \cdots, P_{M_R+M_O}^*)$ is a pure strategy NE of game $\mathcal{G}$. As a consequence, it is impossible for $(P_1^*, \cdots, P_{M_R+M_O}^*)$ to be a pure strategy NE of game $\mathcal{G}$.

Similarly, consider that the player i's strategy profile $(P_1^*, \cdots, P_{M_R+M_O}^*)$ does not satisfy $C2$, which is also a pure strategy NE of $\mathcal{G}$. Then, $i \in \mathcal{K} - \mathcal{K}_{M_R}$, $(P_1^*, \cdots, P_{M_R+M_O}^*) \neq (0, \cdots, 0)$, and $P_i^* \neq 0$. Applying the same analysis method as previously mentioned, it can be concluded that if $(P_1^*, \cdots, P_{M_R+M_O}^*)$ violates $C2$, then it can not be a pure strategy NE of game $\mathcal{G}$.

Hence, if $\beta^P > \beta_{th}^P$ and $\beta^M > \beta_{th}^M$, then it is clear that a strategy profile that fails to meet either $C1$ or $C2$ cannot be a pure strategy NE game $\mathcal{G}$.

Appendix B: Proof of Theorem 3

Let $\mathbf{P}^o = (P_1^o, \cdots, P_{M_R+M_O}^o)$ and $\mathbf{P}^* = (P_1^*, \cdots, P_{M_R+M_O}^*)$ represent the optimal scheme to Problem 2 and a pure strategy NE of game $\mathcal{G}$, respectively. It follows that for an arbitrary player i

$$\begin{aligned}u_i\left(\mathbf{P}^*\right) = U\left(\mathbf{P}^*\right) &= \sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m^* P_{m'}^*}\rho_{mm'}\\ u_i\left(\mathbf{P}^o\right) = U\left(\mathbf{P}^o\right) &= \sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m^o P_{m'}^o}\rho_{mm'}.\end{aligned} \tag{B. 1}$$

Furthermore, according to the condition (i), the strategy profile $\left(P_i^o, P_{-i}^*\right)$ satisfies $C1$ and $C2$, and then it can be obtained that

$$\begin{aligned}u_i\left(P_i^o, P_{-i}^*\right) &= \sum_{m=1,m\neq i}^{M_R+M_O} \sum_{m'=1,m'\neq i}^{M_R+M_O} \sqrt{P_m^* P_{m'}^*}\rho_{mm'}\\ &+ \sum_{m=1}^{M_R+M_O} \sqrt{P_m^* P_i^o}\rho_{mi} + \sum_{m'=1,m'\neq i}^{M_R+M_O} \sqrt{P_i^o P_{m'}^*}\rho_{im'}.\end{aligned} \tag{B. 2}$$

According to the Definition 1, $u_i\left(\mathbf{P}^*\right) \geq u_i\left(P_i^o, P_{-i}^*\right)$, then

$$\left(\sqrt{P_i^*} - \sqrt{P_i^o}\right)\left\{\sum_{m=1}^{M_R+M_O} \sqrt{P_m^*}\rho_{mi} + \sum_{m'=1,m'\neq i}^{M_R+M_O} \sqrt{P_{m'}^*}\rho_{im'}\right\} \geq 0.$$

From the assumption $\rho_{mm'} \geq 0, \forall m, m' \in \{1, \cdots, M_R + M_O\}$ in condition (ii), it can be obtained that

$$P_i^* \geq P_i^o, \forall i \in \mathcal{K}. \tag{B. 3}$$

That is to say,

$$\sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m^* P_{m'}^*}\rho_{mm'} \geq \sum_{m=1}^{M_R+M_O} \sum_{m'=1}^{M_R+M_O} \sqrt{P_m^o P_{m'}^o}\rho_{mm'},$$

which means that $U\left(\mathbf{P}^*\right) \geq U\left(\mathbf{P}^o\right)$. Since assumed that $\mathbf{P}^o$ is an optimal scheme, $U\left(\mathbf{P}^o\right) \geq U\left(\mathbf{P}^*\right)$. Therefore, $U\left(\mathbf{P}^*\right) = U\left(\mathbf{P}^o\right)$ can be obtained, thereby concluding the proof.

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Zhen Wang received the B.Eng., M.Eng. and Ph.D. degree in Electronic Engineering from the University of Electronic Science and Technology of China (UESTC), Chengdu, Sichuan, China, in 2012, 2015, and 2022. She is now a postdoctoral fellow at UESTC and a lecturer at Southwest Petroleum University. Her research interests include integrated sensing and communication, radar signal processing, and communication signal processing.

Longye Wang was born in Gansu Province, P. R. China, on March, 1976. He received the M.S. degree in School of Information Science and Technology of Southwest Jiaotong University, Chengdu, China, in 2005. He pursued his Ph.D. degree the National Key Laboratory of Science and Technology on Communications at UESTC, Chengdu, China. Then he worked as a professor in Tibet University, China. Now he holds the professor position at SWPU, China. His current interests include sequences and its applications.